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PALAZZONE - Scuola Normale Superiore di Pisa

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ABSTRACTS

Shiri Artstein-Avidan

Tel Aviv University

On the two duality transforms for geometric convex functions

We will discuss some recent results, and their classical counterparts, concerning the two order reversing transforms on the class of lower semi continuous non-negative convex functions satisfying f(0)=0.

Gennadiy Averkov

Universität Magdeburg

Analytic and discrete aspects of the covariogram problem

The covariogram of a convex body K in the d-dimensional Euclidean space is a function associating to each vector x the volume of the intersection of K and $K+x$. The covariogram problem (introduced by Matheron) asks about reconstruction of K from the covariogram of K. We discuss the covariogram problem and its discrete version for lattice-convex sets, which was introduced independently by Gardner, Gronchi and Zong and Daurat, Gérard and Nivat.

Imre Bárány

Alfréd Rényi Institute of Mathematics, Budapest - University College London

Jarnik's convex lattice n-gon for non-symmetric norms

Joint work with N. Enriquez

What is the minimum perimeter of a convex lattice n-gon? This question was answered by Jarnik in 1926. We solve the same question in the case when perimeter is measured by a (not necessarily symmetric) norm. We also show that, after suitable normalization, the minimizing convex lattice n-gons have a limit shape.

Gabriele Bianchi

Università di Firenze

Some known results and open problems related to the covariogram

The covariogram g_A of a compact set $A \subset \mathbb{R}^n$ is the function that to each $x \in \mathbb{R}^n$ associates the volume of $A \cap (A + x)$. It coincides with the convolution of the characteristic function of A with the characteristic function of the reflection of A in the origin. We will review some known results and open problems regarding three questions: Does g_A determine A?; Which functions are covariograms?; Which geometric properties of A can be explicitly read in g_A? We will also present some known results (related to the first question) regarding the zero set of the Fourier transform (in complex variables) of the characteristic function of a convex set.

Chiara Bianchini

Université Henri Poincaré, Nancy

Polygonal solutions for a shape optimisation problem

Joint work with Antoine Henrot

We look for the minimizers of the functional $J_\lambda(\Omega) = \lambda|\Omega| - P(\Omega)$ among planar convex domains constrained to lie into a given ring. We prove that, according to the values of the parameter λ, the solutions are either a disc or a polygon. In this last case, we describe completely the polygonal solutions by reducing the problem to a finite dimensional optimization problem.

Sergey Bobkov

University of Minnesota

Information-theoretic extensions in High-dimensional Convex Geometry

Joint work with M. Madiman

In the talk we discuss entropic formulations and extensions of classical geometric results, including the reverse Brunn-Minkowski inequality (due to V.Milman).

Dario Cordero-Erausquin

Université Pierre et Marie Curie, Paris 6

Asymmetric covariance inequalities of Brascamp-Lieb type

The Brascamp-Lieb inequality is a Poincaré type inequality for the variance of a function with respect to a log-concave measure. We present extensions of this inequality, which allows to bound the covariance of two functions, where we replace the L^2 norm of gradients by L^p-L^q norms.

Paolo Dulio

Politecnico di Milano

Reconstruction of twisted polytopes and applications

Joint work with Carla Peri

As it is well known, crystal shapes can be grouped on the basis of their external (i.e. macroscopically visible) symmetry features into seven systems of three dimensional patterns, namely *cubic*, *tetragonal*, *hexagonal*, *trigonal*, *orthorhombic*, *monoclinic*, and *triclinic*.

This external shape reflects the characteristic symmetry of the microscopic pattern of atomic arrangement in the various crystals, and can be well approximated by polyhedral forms. A combinations of forms belonging to a same crystal class may result in a *crystal cluster*, a formation that consists of a number of single-terminated crystals, each adhering to a common base. It turns out that, in view of a tomographic reconstruction, the external structure of a crystal cluster can be well approximated by special unions of different polyhedra.

This motivates the study of clusters of convex polytopes, mutually intersecting according to a *twisting* requirement. Uniqueness results can be determined by means of suitable sets of $n-1$ dimensional X-rays, both in the n dimensional Euclidean space $\mathbb{R}^n$ and in the integer lattice $\mathbb{Z}^n$, for all $n \geq 2$. In particular, from data concerning 1-dimensional X-rays one can derive uniqueness results for cluster of twisted polytopes whose facets are parallel to the given directions

Dan Florentin

Tel Aviv University

Order isomorphisms in windows

We characterize order preserving transforms on the class of lower-semi-continuous convex functions which are defined on a convex subset of $\mathbb{R}^n$ (a "window"), and some of its variants. To this end we investigate convexity preserving maps on subsets of $\mathbb{R}^n$. We prove that in general an order isomorphism is induced by a special convexity preserving point map on the epi-graph of the function. In the case of non-negative convex functions on K, where $0 \in K$ and $f(0) = 0$, one may naturally partition the set of order isomorphisms into two classes; we explain the main ideas behind these results. The talk is based on joint work with S. Artstein and V. Milman.

Richard J. Gardner

Western Washington University, USA

Operations between sets or functions

Joint work with Daniel Hug and Wolfgang Weil

The talk is a report on a long-term project, still in progress, with Daniel Hug and Wolfgang Weil. The general goal is to achieve a proper understanding of the fundamental nature of operations between sets or functions in geometry. For example, a corollary of more general results is that, with trivial exceptions, any operation that maps pairs of compact convex sets in $\mathbb{R}^n$ to compact convex sets in $\mathbb{R}^n$ and which is continuous with respect to the Hausdorff metric, associative, and $GL(n)$-contravariant, must coincide with L_p addition for some $1 \leq p \leq \infty$ when restricted to the origin-symmetric sets.

Apostolos Giannopoulos

University of Athens

On the distribution of the ψ_2-norm of linear functionals on isotropic convex bodies

It is known that every isotropic convex body K in $\mathbb{R}^n$ has a "subgaussian" direction with a constant which is logarithmic in the dimension. We establish the existence of subgaussian directions for K with constant $r = O(\sqrt{\log n})$. This is the best known estimate and follows from the upper bound

$$|\Psi_2(K)|^{1/n} \leqslant \frac{c\sqrt{\log n}}{\sqrt{n}}$$

for the volume of the body $\Psi_2(K)$ with support function

$$h_{\Psi_2(K)}(\theta) := \sup_{2 \leqslant q \leqslant n} \frac{\|\langle \cdot, \theta \rangle\|_q}{\sqrt{q}}.$$

We also introduce the function $\psi_K(t) := \sigma\left(\{\theta \in S^{n-1} : h_{\Psi_2(K)}(\theta) \leqslant ct\sqrt{\log n} L_K\}\right)$ and we discuss lower bounds for $\psi_K(t)$, $t \geqslant 1$. The exact behavior of this function is closely related to several open questions; an example is the problem of giving a sharp upper bound for the mean width of an isotropic convex body in terms of the dimension.

Peter Gritzmann

Technische Universität München

On Gravity Bodies and Power Diagrams

Joint work with Andreas Brieden

We study weighted clustering problems in Minkowski spaces under balancing constraints with a view towards separation properties. The most prominent example in our context is that of the consolidation of farmland.

First, we introduce the gravity polytopes and more general gravity bodies that encode all feasible clusterings and indicate how they can be utilized to develop efficient approximation algorithms for quite general hard to compute objective functions. Then we show that their extreme points correspond to strongly feasible power diagrams, certain specific cell complexes, whose defining polyhedra contain the clusters, respectively. Further, we characterize strongly feasible centroidal power diagrams in terms of the local optima of some ellipsoidal function over the gravity polytope. The global optima can also be characterized in terms of the separation properties of the corresponding clusterings.

Peter Gruber

Technische Universität Wien

Normal Bundles of Convex Bodies

The normal bundle of an o-symmetric, smooth and strictly convex body C in $\mathbb{E}^d$ can be represented by a closed convex cone $\mathcal{N}_C$ in $\mathbb{E}^{d^2}$. In this lecture properties of the cone $\mathcal{N}_C$ and their relations to properties of C are studied.

We begin with a criterion for a cone $\mathcal{N}$ in $\mathbb{E}^{d^2}$ to be a vector bundle cone $\mathcal{N}_C$. Next, it will be shown that the cone $\mathcal{N}_C$ has dimension between $\frac{1}{2}d(d+1)$ and d^2. The dimension is $\frac{1}{2}d(d+1)$ if and only if C is an ellipsoid. With increasing dimensions of $\mathcal{N}_C$ the ellipsoidal character of C decreases. For generic C the dimension of $\mathcal{N}_C$ is d^2. If the cone $\mathcal{N}_C$ has the symmetry properties to coincide with its polar, its transpose or its polar transpose, then, if $d \geq 3$, the body C is a ball or an ellipsoid. For $d = 2$ Radon discs have to be considered too. Finally, there is a one-to-one correspondence between a family of particular faces of $\mathcal{N}_C$ and the planar shadow boundaries of C under parallel illumination.

Christoph Haberl

Universität Salzburg

The even Orlicz-Minkowski problem

Joint work with Erwin Lutwak, Deane Yang, and Gaoyong Zhang

One of the centerpieces of the classical Brunn-Minkowski theory is the Minkowski problem. It concerns the existence of convex hypersurfaces whose Gauss curvature (possibly in a generalized sense) is prescribed as a function of the outer unit normals. The L_p Minkowski problem extends this classical question and was critical for the proofs of affine Sobolev inequalities. We present generalizations of the L_p Minkowski problem which are part of a new Orlicz-Brunn-Minkowski theory

Martin Henk

Universität Magdeburg

Roots of Steiner polynomials

For two convex bodies K, E of the n-dimensional Euclidean space and a non-negative real number λ, the volume of the Minkowski sum $K + \lambda E$ is a polynomial of degree n in λ, known as the (relative) Steiner polynomial of K (with respect to E).

Regarding this (geometric) polynomial as a polynomial in a complex variable we are interested in geometric properties of its roots, e.g., their location in the complex plane (depending on the involved bodies and dimension), size, relation to other functionals and characterization of (families of) convex bodies by mean of properties of the roots. In this talk I will survey on recent results on this topic, which had its starting point in a problem posed by Teissier in 1982 in the context of Algebraic Geometry.
(Based on joint works with Maria Hernandez Cifre and Eugenia Saorin Gomez)

María de los Angeles Hernández Cifre

Universidad de Murcia

Successive radii of convex bodies

Joint work with Bernardo González

For a convex body K of the n-dimensional Euclidean space, we define the successive outer and inner radii, denoted respectively by $\mathrm{R}_i(K)$ and $\mathrm{r}_i(K)$, $i = 1, \ldots, n$, in the following way: $\mathrm{R}_i(K)$ is the smallest radius of a solid cylinder with i-dimensional spherical cross section containing K, whereas $\mathrm{r}_i(K)$ is the radius of the greatest i-dimensional ball contained in K. These measures generalize the well-known functionals diameter, minimal width, circumradius and inradius of K, and can be also defined via the circunradius/inradius of suitable projections/sections of the convex body K.

In this talk I will present a survey on recent results on this topic, showing for instance, the relation between the successive radii with other relevant geometric measures (e.g., the intrinsic volumes), their behavior with respect to the Minkowski addition of convex bodies, or families of convex sets for which their values can be computed.

Daniel Hug

Universität Karlsruhe

Random mosaics in high dimensions

Previously, we considered asymptotic results for random tessellations where some bound for a geometric functional (like the volume) goes to infinity. These results concern random polytopes which arise as particular cells of random tessellations and are related to isoperimetric and stability problems in geometry. (joint work with Rolf Schneider)

There are now also some new investigations of distributional properties of the volume (say) of certain random cells when the dimension of the space is increasing. In this asymptotic framework, Poisson Voronoi and hyperplane tessellations exhibit a surprisingly different behaviour. (joint work with Julia Hörrmann)

Dan Klain

University of Massachusetts Lowell

Some convergence theorems for Steiner symmetrization

Steiner symmetrization is a rearrangement process that preserves volume and convexity of a set, while decreasing surface area. It has long been known that taking limits of iterated Steiner symmetrizations can be used to transform a compact convex set into a Euclidean ball of the same volume. Transformations of this kind were used by Jakob Steiner to give a classical and highly intuitive proof of Brunn's inequality, along with related isoperimetric inequalities. This talk will address convergence questions for infinite sequences of Steiner symmetrizations. Some recent results about convergence will be described and related open questions posed. Some of the new results presented in this talk come from joint work with G. Bianchi, E. Lutwak, D. Yang, and G. Zhang.

Alexander Koldobsky

University of Missouri-Columbia

Stability in volume comparison problems

We extend the hyperplane inequality in dimensions up to 4 to the setting of arbitrary measures in place of the volume. To prove this, we establish stability in the affirmative part of Zvavitch's generalization of the Busemann-Petty problem to arbitrary measures. Then we discuss different stability estimates in similar volume comparison problems.

Monika Ludwig

Technische Universität Wien

Valuations on Function Spaces

Let F be a space of real valued functions, for example, the Sobolev space $W^{1,1}(\mathbb{R}^n)$ and let $\mathbb{A}$ be an abelian semi-group. A function $\mathrm{z} : F \to \mathbb{A}$ is called a *valuation* if

$$\mathrm{z}(f \vee g) + \mathrm{z}(f \wedge g) = \mathrm{z}(f) + \mathrm{z}(g)$$

for all $f, g \in F$, where $f \vee g$ denotes the pointwise maximum and $f \wedge g$ the pointwise minimum of f and g.

We describe classification results for valuations on function spaces. In particular, we obtain a complete classification of affinely contravariant Minkowski valuations on $W^{1,1}(\mathbb{R}^n)$ and show every such valuation z is given as

$$\mathrm{z}(f) = c\,\Pi\langle f\rangle$$

for all $f \in W^{1,1}(\mathbb{R}^n)$ with a suitable constant $c \geq 0$. Here the convex body $\Pi\langle f\rangle$ is defined via its support function as

$$h(\Pi\langle f\rangle, v) = \frac{1}{2}\int_{\mathbb{R}^n} |v \cdot \nabla f(x)|\, dx$$

for $v \in \mathbb{R}^n$.

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Xi-Nan Ma

East China Normal University

The Gaussian curvature estimates for the convex level sets of harmonic function.

I shall report the lower bounded estimates of the Gaussian curvature for the convex level sets of harmonic function, and study the relation of the curvature of the convex level sets and the height of the harmonic function. We use the maximum principle and support function.

Mathieu Meyer

Université de Marne-La-Vallée, Paris

The convex intersection body of a convex body

We prove that the convex intersection body of a convex body is convex, we study some of its properties and ask a few open problems.

Vitali Milman

Tel Aviv University

Some new results on "asymptotic" convexity
(by our young generation, following D. Faifman, L. Rotem, A. Segal and B. Slomka)

I will provide results in three subjects:

1. Duality through section/projection exchange (following Milman-Segal-Slomka)

2. Mean-width of log-concave functions, generalization of Urysohn inequality and Low M*-estimate (following L.Rotem)

3. Projection width and Alvarez-Paiva-type theorems (following D.Faifman).

Vladimir I. Oliker

Emory University, Atlanta

Convexity and mass transport methods for solving problems in optics

We discuss the problem of determining a system of two refractive interfaces transforming a plane wavefront of a given shape and radiation intensity into a coherent output plane wavefront with prescribed output position, shape and intensity. In geometrical optics approximation the analytic formulation of this problem requires construction of maps with controlled Jacobian. In this talk I will outline a geometric convexity approach for re-formulating the problem in certain associated measures and defining weak solutions. Existence and uniqueness of weak solutions in Lipschitz classes is established via a connection to optimal mass transport.

Alain Pajor

Université de Marne-la-Vallée, Paris

Low dimensional faces of high dimensional random polytopes

We will survey recent results on centrally symmetric random polytopes in high dimension, in particular regarding the low dimensional structure of faces and the property of neighborliness (centrally); starting from the work of D. Donoho for Gaussian polytopes to the recent joint works with R.Adamczak, A. Litvak and N. Tomczak-Jaegermann in the general case of polytopes generated by random points from a convex body.

Aldo Pratelli

Università of Pavia

Solution to the Auerbach Conjecture

We will discuss about the Santalo's problem of determining the convex planar set minimizing the area among those having all the bisecting chords bigger than a given constant. By "bisecting chord" we mean any segment which divides the convex set in two parts of half area. As it has been proved in last two years, the solution is the so-called Auerbach Triangle, and not -as one could imagine, and as Santalo's himself conjectured- the disk. We will describe the problem, giving a rough idea of the proof, and we will conclude by showing some features of the general, and completely open, problem without the convexity constraint.

Paolo Salani

Università di Firenze

The Hot Spot of a Convex Set

The talk is based on the paper "L. Brasco, R. Magnanini and P. Salani, *The location of the hot spot in a grounded convex conductor*, (2010)", available at http://arxiv.org/abs/1012.4742.

Consider a convex heat conductor Ω having (positive) constant initial temperature while its boundary is constantly kept at zero temperature. A *hot spot* $x(t) \in \Omega$ is a point at which the temperature $u(x,t)$ attains its maximum at time t.

If Ω is convex, it is well-known by a result of Brascamp and Lieb that $\log u(x,t)$ is concave (and analytic) in x, and this implies that for every $t > 0$ there is a unique hot spot $x(t) \in \Omega$. A classical result of Varadhan's tells us where $x(t)$ is located for small times:

$$\operatorname{dist}(x(t), \partial\Omega) \to r_\Omega \quad \text{as } t \to 0^+, \tag{1}$$

where

$$r_\Omega = \max\{\operatorname{dist}(y, \partial\Omega) \,:\, y \in \overline{\Omega}\}$$

is the *inradius* of Ω.

For large times instead, we know (as noticed by Sakaguchi) that $x(t)$ must be close to the maximum point x_∞ of the first Dirichlet eigenfunction ϕ_1 of $-\Delta$:

$$x(t) \to x_\infty \quad \text{as } t \to \infty. \tag{2}$$

While, thanks to (1), it is relatively easy to locate $x(t)$ for small times by geometrical means, (2) does not give much information: locating either $x(t)$ or x_∞ has more or less the same difficulty. I present two geometrical methods to estimate the location of $x(t)$ and/or x_∞, based on two kinds of arguments. The former is based on tools of convex analysis and exploits some properties of the polar of a convex set. The latter argument relies instead on the moving plane method and the *Alexandrov's reflection principle*. Based on this, for a convex body $\mathcal{K}$ we can define another convex set $\heartsuit(\mathcal{K})$ — the *heart* of $\mathcal{K}$ — such that $x(t) \in \heartsuit(\mathcal{K})$ for every $t > 0$ (in fact, we will prove that $\mathcal{K} \setminus \heartsuit(\mathcal{K})$ cannot contain the hot spot $x(t)$ for any $t > 0$). I will present some properties of $\heartsuit(\mathcal{K})$ and will suggest some open (maybe easy) questions.

Eugenia Saorín Gómez

Universität Magdeburg

A characterization of some mixed volumes via the Brunn-Minkowski inequality

Joint work with A. Colesanti and D. Hug

We consider a functional $\mathcal{F}$ of the form:

$$\mathcal{F}(K) = \int_{\mathbb{S}^{n-1}} f(u)\, d\mathrm{S}(K,u)\,,$$

where $f \in \mathcal{C}(\mathbb{S}^{n-1})$, K is a convex body and $\mathrm{S}(K,\cdot)$ is the area measure of K. We prove that if $\mathcal{F}$ satisfies a Brunn–Minkowski type inequality and either $\mathcal{F}$ is even or $f \in W^{1,2}(\mathbb{S}^{n-1})$, then f is the support function of a convex body, i.e. $\mathcal{F}$ is a mixed volume.

Franz Schuster

Technische Universität Wien

The Petty projection inequality and beyond

A classical result of Cauchy states that the Euclidean surface area of the boundary of a convex body can be computed by averaging the areas of the shadows cast by the body over all possible directions. In the early 1970s Petty discovered that it is possible to do this averaging differently and obtain an affine invariant notion of surface area instead. This invariant could be interpreted as the volume of the polar projection body associated with the convex body. Petty also proved that this affine invariant satisfies a fundamental affine isoperimetric inequality, now known as Petty's projection inequality, that is far stronger than the classical Euclidean isoperimetric inequality.

The Petty projection inequality was the starting point for recent remarkable results in the area of affine isoperimetric and analytic inequalities: About a decade ago Lutwak, Yang, and Zhang established an important L_p version of Petty's projection inequality for the (symmetric) L_p analogue of the projection operator. This L_p extension is the core of a full affine analogue of the classical Pólya–Szegö principle by Cianchi, Lutwak, Yang and Zhang. Among many applications of this new affine symmetrization principle are sharp affine Sobolev, Moser–Trudinger and Morrey–Sobolev inequalities.

In this talk we will survey these results and discuss new material based on advances in valuation theory by Ludwig which led to a stronger asymmetric version of the L_p Petty projection inequality (joint work with C. Haberl) and a new asymmetric affine Pólya–Szegö principle (joint work with C. Haberl and J. Xiao). These most recent developments culminated in the work of Lutwak, Yang and Zhang on affine isoperimetric inequalities for Orlicz projection bodies.

Carsten Schütt

Universität Kiel

Mahler's conjecture and curvature

Joint work with Elisabeth Werner and Shlomo Reisner

Let K be a convex body in $\mathbb{R}^n$ with Santaló point at 0. We show that if K has a point on the boundary with positive generalized Gauß curvature, then the volume product $|K||K^\circ|$ is not minimal. This means that a body with minimal volume product has Gauß curvature equal to 0 almost everywhere and thus suggests strongly that a minimal body is a polytope.

Boaz Slomka

Tel Aviv University

Order-isomorphisms for cones and ellipsoids

Joint work with Shiri Artstein-Avidan

We determine the general form of order-isomorphisms associated to cones in $\mathbb{R}^n$, and give additional conditions under which such maps must be affine-linear. As a consequence, we obtain a characterization of the duality mapping defined on ellipsoids.

Nicole Tomczak-Jaegermann

University of Alberta

Geometry of log-concave Ensembles of random matrices

The talk is based on a series of joint papers by Radoslaw Adamczak, Rafal Latala, Alexander Litvak, Alain Pajor, and the speaker.

We discuss new tail estimates for order statistics for norms of projections of isotropic log-concave vectors in $\mathbb{R}^N$. As an immediate consequence we obtain a uniform version of Paouris' deviation inequality for norms of projections of such vectors. This is used in the study of random matrices with independent isotropic log-concave rows to prove uniform bounds for the operator norm of their submatrices with k rows and m columns. Such uniform bounds in particular allow to control some related concentration inequalities.

Beatrice-Helen Vritsiou

University of Athens

A measure of orthogonality on isotropic convex bodies

Lutwak, Yang and Zhang have proved that, for every $q \geq 1$, the minimum of the quantity

$$Y_q(K,M) := \left(\int_K \int_M |\langle x,y\rangle|^q dy\, dx \right)^{1/q}$$

over all compact sets K and M of Lebesgue measure 1 in $\mathbb{R}^n$ is attained when $K = M = D_n$, the Euclidean ball of volume 1. Note that $Y_q(D_n, D_n) \simeq \sqrt{qn}$ for all $1 \leq q \leq n$. We discuss upper bounds for $Y_q(K) := Y_q(K,K)$ and related quantities in the case where K is an isotropic convex body in $\mathbb{R}^n$.

Wolfgang Weil

Universität Karlsruhe

Mean section bodies

Joint work with P. Goodey

The k-th mean section body $M_k(K)$ of a convex body K is the Minkowski average of all k-dimensional sections of K (intersections with k-dimensional affine flats). Whereas $M_d(K) = K$ and $M_1(K)$ is always a ball, the case $2 \leq k \leq d-1$ raises the interesting question, whether $M_k(K)$ determines K uniquely. For $k = 2$ (i.e. planar sections), uniqueness was shown in Goodey-W. (1992), up to translations. The translational restriction was later removed by Goodey (1998) who also proved uniqueness for $2 \leq k \leq d-1$ and centrally symmetric bodies. Now we can show the uniqueness result in general.

Elisabeth Werner

Case Western Reserve University, Cleveland

How often is a random quantum state k–entangled?

Joint with S. Szarek and K. Zyczkowski

The set of trace preserving, positive maps acting on density matrices of size d forms a convex body. We investigate its nested subsets consisting of k–positive maps, where $k = 2, \ldots, d$. Working with the measure induced by the Hilbert-Schmidt distance we derive asymptotically tight bounds for the volumes of these sets.

Deane Yang

Polytechnic University, Brooklyn, New York

Affine and Orlicz information theory

Ideas from affine and Orlicz geometric analysis are used to define generalized information theoretic invariants and establish sharp inequalities for them.

Vladyslav Yaskin

University of Alberta

A problem of Klee on inner section functions of convex bodies

Joint work with R. Gardner, D. Ryabogin and A. Zvavitch.

In 1969, Vic Klee asked whether a convex body is uniquely determined (up to translation and reflection in the origin) by its inner section function, the function giving for each direction the maximal area of sections of the body by hyperplanes orthogonal to that direction. We answer this question in the negative by constructing two infinitely smooth convex bodies of revolution about the x_n-axis in $\mathbb{R}^n$, $n \geq 3$, one origin symmetric and the other not centrally symmetric, with the same inner section function. Moreover, the pair of bodies can be arbitrarily close to the unit ball.

Artem Zvavitch

Kent State University

Intersection bodies and some generalizations of the Busemann's Theorem

The notion of an intersection body of a star body was introduced by Erwin Lutwak: K is called the intersection body of L if the radial function of K in every direction is equal to the (d-1)-dimensional volume of the central hyperplane section of L perpendicular to this direction.

The notion turned out to be quite interesting and useful in Convex Geometry and Geometric Tomography. Still there are the number of open questions. For example: it is easy to see that the intersection body of a ball is again a ball. E. Lutwak asked if there is any other star-shaped body that satisfy this property.

Busemann's theorem states that the intersection body of an origin-symmetric convex body is also convex. But what can we say if the body is not convex?

In this talk we provides some partial answers to those questions, we will present a solution to local version for Lutwak's question and link it to new generalization of Busemann's theorem for quasi-convex bodies. (the talk is based on joint works with A. Fish, J. Kim, F. Nazarov, D. Ryabogin and V. Yaskin)

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