EIGENVECTORS AND BEST RANK K APPROXIMATION FOR BINARY FORMS

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Abstract

I start this talk with the definition of best rank k approximation problem for the case of matrices and symmetric matrices. With this purpose I define the distance function and its critical points. Moreover, I give a geometric interpretation for each of these cases in terms of the Segre and Vernoese variety and the eigenvectors. In the case of symmetric matrices I give explicitly Eckart-Young theorem. For an arbitrary real algebraic variety can be defined the EDdegree, which is the number of critical points of the euclidean distance function. (More details in [1] and [2]).

I give also the definition of symmetric tensors in terms of homogeneous polynomial. I explain the best rank 1 approximation problem by defining the distance function, its critical points and the relation with the eigenvectors. Moreover I point out the differences between matrices and tensors.

Then I focus on the special case of binary forms (homogeneous polynomials in two variables $f(x, y)$). In this case the eigenvectors of f can be defined as the roots of the differential operator $D(f) = yf_x - xf_y$. Geometrically the binary forms of rank 1 can be seen as points on the rational normal curve (C_d).

Finally, concerning binary forms, I give some new results collected in [3]. We give an interpretation in terms of tangent and normal space of the critical points of the distance function restricted to the $k-$secant variety of the rational normal curve ($\sigma_k C_d$). Moreover, we define the singular space H_f as the hyperplane killed by the differential operator $D(f)$. We also prove that the critical points of the form $\sum_{i=1}^{k} l_i^d$, where l_i are linear forms, of the distance function restricted to $\sigma_k C_d$ belong to the singular space H_f. Furthermore, for the case $k = 1$ we prove that H_f coincide with the span of the eigenvectors of f.

References

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[2] G. Ottaviani, R. Paoletti, A Geometric Perspective on the Singular Value Decomposition, Rend. Istit. Mat. Univ. Trieste, Volume 47, 107- 125, 2015.

[3] G. Ottaviani, A. Tocino, *Best rank k approximation for binary forms*, on preparation.