A boundary integral equation method for linear elastodynamics problems in unbounded domains

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We consider transient 3D elastic wave propagation problems in unbounded isotropic homogeneous media, which can be reduced to corresponding 2D ones. This is the case, for example, of problems defined on the exterior of a bounded rigid domain, which are invariant in one of the cartesian directions. The linear elastodynamics problem that characterizes small variations of a displacement field $\mathbf{u}$ in a medium Ω^e having the above properties, caused by a body force $\mathbf{f}$ and a Dirichlet datum $\mathbf{g}$, is defined by the following system:

$$\begin{cases} \rho \dfrac{\partial^2 \mathbf{u}}{\partial t^2}(\mathbf{x},t) - (\lambda+\mu)\nabla(\operatorname{div}\mathbf{u})(\mathbf{x},t) - \mu\,\nabla^2\mathbf{u}(\mathbf{x},t) & = \mathbf{f}(\mathbf{x},t) \quad (\mathbf{x},t) \in \Omega^e \times (0,T) \\ \mathbf{u}(\mathbf{x},t) & = \mathbf{g}(\mathbf{x},t) \quad (\mathbf{x},t) \in \Gamma \times (0,T) \\ \mathbf{u}(\mathbf{x},0) & = \mathbf{0} \qquad\quad \mathbf{x} \in \Omega^e \\ \mathbf{u}_t(\mathbf{x},0) & = \mathbf{0} \qquad\quad \mathbf{x} \in \Omega^e, \end{cases} \tag{1}$$

where $\Gamma = \partial\Omega^e$, $\rho > 0$ is the constant material density, $\lambda > 0$ and $\mu > 0$ are the Lamé constants.

By applying a classical Helmholtz decomposition, $\mathbf{u} = \nabla\varphi_P + \mathbf{curl}\,\varphi_S$ and $\mathbf{f} = \nabla \mathrm{f}_P + \mathbf{curl}\,\mathrm{f}_S$, we split the elastic vector equation into a couple of scalar wave equations describing, respectively, the propagation of P-waves (Primary or longitudinal) and S-waves (Secondary or transverse):

$$\begin{cases} \dfrac{\partial^2 \varphi_P}{\partial t^2}(\mathbf{x},t) - v_P^2 \nabla^2 \varphi_P(\mathbf{x},t) & = \dfrac{1}{\rho}\mathrm{f}_P(\mathbf{x},t) \qquad v_P = \sqrt{\dfrac{\lambda+2\mu}{\rho}} \\ \dfrac{\partial^2 \varphi_S}{\partial t^2}(\mathbf{x},t) - v_S^2 \nabla^2 \varphi_S(\mathbf{x},t) & = \dfrac{1}{\rho}\mathrm{f}_S(\mathbf{x},t) \qquad v_S = \sqrt{\dfrac{\mu}{\rho}}. \end{cases} \tag{2}$$

The two equations are coupled by the Dirichlet boundary condition $\nabla\varphi_P + \mathbf{curl}\,\varphi_S = \mathbf{g}$ on Γ.

We reformulate (2) in terms of its associated space-time BIE representation. For the discretization of the new proposed approach, we combine a BDF2 Lubich time convolution quadrature formula with a classical space collocation method based on piece-wise linear approximation of the unknowns. Several numerical results, including comparisons with the vector space-time boundary integral formulation approach, are presented and discussed.

References

[1] S. Falletta and G. Monegato L. Scuderi, Two boundary integral equation methods for linear elastodynamics problems on unbounded domains, *Computers and Mathematics with Applications*, DOI:10.1016/j.camwa.2019.06.017.