# $C^2$ Cubic Splines on Triangulations

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Splines on triangulations have widespread applications in many areas, ranging from finite element analysis and physics/engineering applications to computer graphics and entertainment industry. Piecewise polynomials of degree 3 of class $C^2$ are very appealing because they couple the low degree with a smoothness which allows to efficiently address several problems.

When dealing with a general triangulation, to obtain $C^2$ smoothness in a stable manner, one must use polynomials of degree 9 on each triangle. An alternative is to use lower-degree macro-elements that subdivide each triangle into a number of subtriangles (or more generally subdomains). The most common macro-structures are the Powell-Sabin split and the Clough-Tocher split. The minimum degree to get $C^2$ smoothness is 5 on the Powell-Sabin split, while piecewise polynomials of degree 6 are needed to achieve $C^2$ smoothness on the Clough-Tocher split, see [1].

Locally supported basis functions are necessary both for efficient computation and for optimal approximation power of the considered spline space. Simplex splines are one of the most elegant generalizations of univariate B-splines to the multivariate setting. They can be interpreted as the density function of a simplex shadow. This beautiful geometric construction allows to easily derive properties such as smoothness and recursion, knot insertion and degree elevation formulas.

In this talk we consider a $C^2$ cubic spline space that can be defined on any given triangulation suitably refined and we construct a simplex-spline basis for it. This space ensures full approximation power and any element of the space admits a local construction. Besides computational efficiency, the provided simplex-spline basis possesses all the most important properties we wish for when dealing both for geometric modelling and approximation. More precisely, our basis enjoys the following properties:

- Nonnegativity, partition of unity and local support;
- Representation in terms of a geometrically meaningful control polygon and geometric interpretation of smoothness conditions between adjacent triangles.

The talk is based on a joint work with Tom Lyche and Hendrik Speleers.

## References

[1] Lai, M-J. and Schumaker, L. L. *Spline Functions on Triangulations*. Cambridge University Press (2007).

[2] Micchelli, C. A. On a numerically efficient method for computing multivariate B-splines, in: *Multivariate Approximation Theory* (W. Schempp and K. Zeller, eds.), Birkhauser Verlag, Basel, pp. 211-248 (1979).