

A new family of Hermite-Birkhoff spline quasi-interpolation schemes and their application to derive high order continuous extensions for Gauss Runge-Kutta methods.

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Symmetric one step Hermite-Obrechhoff methods (BSHO) [1] are multi-derivative methods for ODEs admitting an intrinsic spline extension. Such peculiarity allows us to give them a dual interpretation in the context of discrete spline quasi-interpolation (QI) requiring Hermite-Birkoff data at the spline knots. The related QI operators have maximal approximation order and are projectors in the considered spline space whose smoothness is settable between R and $2R - 1$, being $2R$ the spline degree. As an application, they have been used for an efficient computation of a high order continuous extension of the numerical solution produced by a Gauss-Runge-Kutta scheme. As for all superconvergent schemes, although the formulas for the s stage methods are of order $2s$ at the mesh points, they have just *uniform* order s , since their classical continuous extension—a spline usually represented in local polynomial form—has just order s . For example, COLSYS [2]—a popular code for the solution of BVPs based on these schemes—uses Gauss-Legendre formulas of quite high order for which the uniform order of approximation is half the order at the mesh points. Using the new QI schemes we efficiently compute a continuous extension of order $2s$. This is a joint work with Alessandra Sestini.

References

- [1] F. Mazzia, A. Sestini, On a class of Conjugate Symplectic Hermite-Obreshkov one-step methods with continuous spline extension, *Axioms* (2018), **7**, 58, 1–18.
- [2] U. Asher, J. Christiansen, R.D. Russell, Algorithm 569: COLSYS: Collocation software for Boundary-Value ODEs, *ACM Transactions of Mathematical Software*, **7**, 223–229.