Error estimates for splines of arbitrary smoothness in IGA

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ABSTRACT

Classical error estimates in spline approximation read as follow [1]: for any $u \in H^r(0,1)$, and any knot vector τ, there exists a spline $s \in \mathcal{S}^k_{p,\tau}$ such that

$$\|u - s\| \leq C(p,k,r)h^r\|u^{(r)}\|, \quad p \geq r-1,$$

where h denotes the maximal knot distance of τ and $\|\cdot\|$ is the L^2 norm. Here the "constant" $C(p,k,r)$ is independent of h, but depends on the degree p, the smoothness of the spline k, and the Sobolev regularity r. In [2] a representation in terms of Legendre polynomials was exploited to provide a constant $C(p,k,r)$ of the form $C/(p-k)^r$ for spline spaces of degree $p \geq 2k+1$ and smoothness $\mathcal{C}^k$.

In this talk [3] we extend the result of [2] in two ways: (i) we prove that the constant $C(p,k,r)$ is bounded by $C/(p-k)^r$ for any $-1 \leq k \leq p-1$ and (ii) we provide an explicit upper bound of the unknown constant C. The presented error estimates indicate that smoother spline spaces exhibit a better approximation behavior per degree of freedom, even for low regularity of the functions to be approximated. This is in complete agreement with the numerical evidence found in the literature.

We further discuss the extension of these results to the case of tensor product spline approximation and to isogeometric spline spaces generated by means of a mapped geometry, both in the single-patch and in the multi-patch case.

REFERENCES

[1] Schumaker, L. L. *Spline Functions: Basic Theory, 3rd edn*. Cambridge University Press (2007).

[2] Beiro da Veiga, L., Buffa, A., Rivas, J. and Sangalli, G. *Some estimates for h-p-k-refinement in Isogeometric Analysis.* Numer. Math. (2011) 118: 271.

[3] Sande, E., Manni, C. and Speleers, H. *Explicit error estimates for spline approximation of arbitrary smoothness in isogeometric analysis.* arXiv:1909.03559.