Spectral time solution of differential equations

Luigi Brugnano

Joint work with Pierluigi Amodio and Felice Iavernaro

Abstract

In recent years, a new tool named *discrete line integral* has been successfully exploited to derive several classes of geometric integrators suitable for a wide range of conservative problems. The main instance is given by an energy conserving variant of the well-known Gauss-Legendre collocation methods (see, e.g., [2, 3]). In its simplest formulation, this technique performs a projection of the vector field onto a finite dimensional subspace [6] via a least square approach based on the use of Legendre orthogonal polynomials, leading to a finite-dimensional nonlinear system. This fact suggests their use as spectral methods in time. Indeed, recently this spectral implementation has been used to solve highly-oscillatory [7] and stiffly-oscillatory Hamiltonian problems emerging from the space semi-discretization of Hamiltonian PDEs [4]. Interestingly, the ability of working by using quite large stepsizes, due the availability of a very efficient nonlinear iteration technique to handle the nonlinear systems needed to advance the solution in time [5], makes the spectral approach competitive with standard finite order time-stepping methods, even when applied for solving generic initial value problems [1].

References

[1] P. Amodio, L. Brugnano, F. Iavernaro. Analysis of Spectral Hamiltonian Boundary Value Methods (SHBVMs) for the numerical solution of ODE problems. *Numer. Algor.* (2019) 1–20 `https://doi.org/10.1007/s11075-019-00733-7`

[2] L. Brugnano, F. Iavernaro. *Line Integral Methods for Conservative Problems.* Chapman and Hall/CRC, Boca Raton, FL, 2016.

[3] L. Brugnano, F. Iavernaro. Line Integral Solution of Differential Problems. *Axioms* **7**(2) (2018) article n. 36. `http://dx.doi.org//10.3390/axioms7020036`

[4] L. Brugnano, F. Iavernaro, J.I. Montijano, L. Rández. Spectrally accurate space-time solution of Hamiltonian PDEs. *Numer. Algorithms* **81**, no. 4 (2019) 1183–1202.

[5] L. Brugnano, F. Iavernaro, D. Trigiante. A note on the efficient implementation of Hamiltonian BVMs. *J. Comput. Appl. Math.* **236** (2011) 375–383.

[6] L. Brugnano, F. Iavernaro, D. Trigiante. A simple framework for the derivation and analysis of effective one-step methods for ODEs. *Appl. Math. Comput.* **218** (2012) 8475–8485.

[7] L. Brugnano, J.I. Montijano, L. Rández. On the effectiveness of spectral methods for the numerical solution of multi-frequency highly-oscillatory Hamiltonian problems. *Numer. Algorithms* **81**, no. 1 (2019) 345–376.