MR2840603 14N05 14N20 15A66 51N35
Angelini, Elena [Angelini, Elena^2] (I-FRNZ)
Higher secants of spinor varieties. (English summary)
Boll. Unione Mat. Ital. (9) **4** (2011), *no. 2,* 213–235.

Let $\mathbb{P}^N$ denote the N-dimensional complex projective space, and let $X \subset \mathbb{P}^N$ be an n-dimensional projective variety spanning $\mathbb{P}^N$. For $k \geq 1$, denote by $\sigma_k(X) \subset \mathbb{P}^N$ the k-th self-join of X, that is, the locus of $(k-1)$-dimensional linear subspaces spanned by general k-ples of points of X (the author calls this object the k-secant variety of X; other people prefer calling it the $(k-1)$-st secant variety of X). A dimension count shows that $\dim \sigma_k(X) \leq \min\{k(n+1)-1, N\}$. The difference $\delta_k = \min\{k(n+1)-1, N\} - \dim \sigma_k(X)$ is called the k-defect of X (in another terminology, δ_k is the $(k-1)$-st secant defect of X); X is called k-defective if $\delta_k > 0$.

Recently the problem of k-deficiency (for $k > 2$) was studied for some important series of homogeneous varieties, such as Grassmann varieties and Segre-Veronese varieties. In the paper under review the author explores the case when $X = S_h$ is the spinor variety parametrizing one of the (maximal) families of $(h-1)$-dimensional linear subspaces on a nonsingular $(2h-2)$-dimensional quadric $Q \subset \mathbb{P}^{2h-1}$, so that $n = h(h-1)/2$, $N = 2^{h-1}-1$, and the image of S_h under the Veronese embedding v_2 is the subvariety of the Grassmann variety $G(h-1, 2h-1)$ parametrizing the linear subspaces of the above family. By using the Terracini lemma (according to which the dimension of $\sigma_k(X)$ is equal to that of the linear span of the tangent spaces to X at a general k-ple of points of X), the Pfaffian parametrization of S_h and the Macaulay2 computer algebra system, the author presents an algorithm for computing $\dim \sigma_k(S_h)$, which allows her to find out whether S_h is k-defective for small values of k and h. Three interesting examples of defective spinor varieties are found, viz. S_7 is 3-defective with $\delta_3 = 5$, S_8 is 3-defective with $\delta_3 = 1$, and S_8 is 4-defective with $\delta_4 = 4$. In fact, the author's method allows her to show that a variety is non-defective and to obtain a list of varieties that *might* be defective; then the defects of the suspicious varieties are computed on a case-by-case basis.

Finally, using an induction argument, the author shows that S_7 and S_8 are the *only* 3-defective spinor varieties (it is known that there are no 2-defective spinor varieties). Unfortunately, the methods used in the paper do not seem to explain the geometric reason for the deficiency of these particular spinor varieties and the structure of the respective entry loci.

Fyodor L. Zak

References

1. H. Abo - G. Ottaviani - C. Peterson, *Non defectivity of Grassmannians of planes*, arXiv:0901.2601v1 [math.AG], to appear on Journal of Algebraic Geometry. MR2846677
2. H. Abo - G. Ottaviani - C. Peterson, *Induction for secant varieties of Segre varieties*, Trans. Amer. Math. Soc., **361**, no. 2 (2009), 767–792. MR2452824
3. J. Alexander - A. Hirschowitz, *Polinomial interpolation in several variables*, Journal of Algebraic Geometry, **4**, n. 2 (1995), 201–222. MR1311347
4. E. Angelini, *Varietà secanti alle varietà spinoriali*, Laurea Thesis, Università di Firenze (2009).

5. A. Boralevi - J. Buczynski, *Secants of Lagrangian Grassmannians*, arXiv:1006.1925v1 [math.AG], to appear on Annali di Matematica. MR2861067
6. K. Baur - J. Draisma - W. de Graaf, *Secant dimensions of minimal orbits: computations and conjectures*, Experimental Mathematics, **16**, no. 2 (2007), 239–250. MR2339279
7. M. C. Brambilla - G. Ottaviani, *On the Alexander-Hirschowitz Theorem*, J. Pure Appl. Algebra, **212** (2008), 1229–1251. MR2387598
8. C. Chevalley, *The algebraic theory of spinors*, Columbia University Press, New York (1954). MR0060497
9. L. Chiantini - C. Ciliberto, *Weakly defective, varieties*, Trans. Amer. Math. Soc., **354**, no 1 (2001), 151–178. MR1859030
10. M. V. Catalisano - A. V. Geramita - A. Gimigliano, *Secant varietes of* $\mathbb{P}^1 \times \cdots \times \mathbb{P}^1$ *(n- times) are NOT Defective for* $n \geq 5$", arXiv:0809.1701, to appear on Journal of Algebraic Geometry. MR2762993
11. H. Kaji, *Homogeneous projective varieties with degenerate secants*, Trans. Amer. Math. Soc., **351**, no 2 (1999), 533–545. MR1621761
12. D. R. Grayson - M. E. Stillman, *Macaulay2*, Software system available at http://www.math.uiuc.edu/Macaulay2/.
13. J. M. Landsberg, *The geometry of Tensors with applications*, Book in preparation (2009). MR2865915
14. J. M. Landsberg - J. Weyman, *On secant varieties of Compact Hermitian Symmetric Spaces*, J. Pure Appl. Algebra, **213** (2009), 2075–2086. MR2533306
15. L. Manivel, *On spinor varieties and their secants*, SIGMA 5 (2009) 078, special volume "Elie Cartan and Differential Geometry". MR2529169
16. F. L. Zak, *Tangents and Secants of Algebraic Varieties*, Translations of Mathematical Monographs, **127**. AMS, Providence, RI, 1993. MR1234494

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.