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Waring decompositions and identifiability via Bertini and Macaulay2 software. (English summary)

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Summary: "Starting from our previous papers Angelini et al. (2018c) [MR3720862] and Angelini et al. (2018a) [MR3781595], we prove the existence of a non-empty Euclidean open subset whose elements are polynomial vectors with 4 components, in 3 variables, degrees, respectively, 2, 3, 3, 3 and rank 6, which are not identifiable over $\mathbb{C}$ but are identifiable over $\mathbb{R}$. This result has been obtained via computer-aided procedures suitably adapted to investigate the number of Waring decompositions for general polynomial vectors over the fields of complex and real numbers. Furthermore, by means of the Hessian criterion (Chiantini et al., 2014), we prove identifiability over $\mathbb{C}$ for polynomial vectors in many cases of sub-generic rank."

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.