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On the identifiability of ternary forms. (English summary)
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The identifiability of symmetric tensors is an interesting problem on its own and it is relevant for applications as image reconstruction and machine learning, among others.

A symmetric tensor corresponds to a form (homogeneous polynomial) of some fixed degree, let's say d. A Waring decomposition of a symmetric tensor expresses it as a sum of d-powers of linear forms and the rank of the tensor is the minimal quantity of linear forms for which the decomposition exists. A symmetric tensor is called identifiable when the linear forms appearing in a minimal decomposition are unique, up to order and scale.

The problem of determining whether a given decomposition has minimal cardinality and whether it is unique or not was solved by Sylvester for binary forms (in two variables).

Known methods for detecting identifiability, including the celebrated Kruskal criterion, apply for values of the tensor's rank below the generic rank—which is the rank realized outside a Zariski closed subset of the space of all forms of degree d.

In this paper, working over the complex numbers and using some nice algebraic geometry techniques to deal with forms in three variables, the authors are able to determine the identifiability of a symmetric tensor, for all values of the rank up to the generic one.

Interpreting a linear form as a point of a projective plane, and considering the Veronese map v_d that raises to the d power every linear form, a decomposition of a symmetric tensor corresponds to a finite set A of these points such that the tensor belongs to the linear span of the image $v_d(A)$.

By means of the Hilbert function and a resolution of the ideal of A, an algorithm is given that can guarantee the uniqueness of a given decomposition for the case of degree $d = 8$. It is claimed that, in principle, the same method works for any degree and that with the same approach one could study the case of forms in more than three variables.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.