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Real identifiability vs. complex identifiability. (English summary)
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A tensor with complex entries which may be minimally decomposed into a sum of r simple tensors has *rank* r. If this decomposition is unique, then the tensor is *identifiable*. Most tensors in a particular space have a particular rank, and the tensors with rank smaller than this are said to have *sub-generic* rank. Most tensors of sub-generic rank are expected to be identifiable, so non-identifiable tensors of sub-generic rank are particularly interesting.

A natural question arises when looking at these non-identifiable tensors. Of the various decompositions into sums of complex simple tensors, how many involve only real entries? If there exists only one such decomposition, then we say that the tensor is identifiable over $\mathbb{R}$. It is this question that the authors explore.

Real elliptic normal curves are used throughout. The authors begin by looking at the $\mathbb{P}^3$ case, where real elliptic normal curves are complete intersections of two quadrics. A general point in this space is the intersection of two secant lines to such a curve. Either all four points of intersection between the secant lines and the curve are real, two are real and two are nonreal, or all four are nonreal.

This idea extends by induction in a natural way. Again, a general point in $\mathbb{P}^{2r-1}$ is the intersection of two secant $(r-1)$-spaces to a real elliptic normal curve. There exists an open ball of such points where one secant space intersects the curve in real points and the other secant space intersects the curve with some nonreal points. This corresponds to a tensor which is not identifiable over $\mathbb{C}$, as each secant space corresponds to a different tensor decomposition, but is identifiable over $\mathbb{R}$. *Douglas A. Torrance*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.