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Identifiability for a class of symmetric tensors. (English summary)
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In the paper under review, the authors study an *identifiability* problem of certain symmetric tensors over the field of complex numbers $\mathbb{C}$.

Let T be a symmetric tensor of type $(n+1)\times\cdots\times(n+1)$ (d times), which can be identified with a polynomial of degree d in $n+1$ variables. T is called 'identifiable' if T has a unique Waring decomposition $T=L_1^d+\cdots+L_r^d$ up to trivialities (i.e., permutations of the summands or rescaling of them). This identifiability problem is an interesting question on its own and a very important issue in many applications as well [see, e.g., C. J. Appellof and E. R. Davidson, Anal. Chem. **53** (1981), no. 13, 2053–2056, doi:10.1021/ac00236a025].

The identifiability of a *general* symmetric tensor of subgeneric range (i.e., of rank smaller than the generic rank $\lceil r_{d,n}\rceil, r_{d,n} := \frac{\binom{d+n}{n}}{n+1}$) turns out to be true with a finite list of exceptions by [L. Chiantini, G. M. Ottaviani and N. Vannieuwenhoven, Trans. Amer. Math. Soc. **369** (2017), no. 6, 4021–4042; MR3624400]. But, the identifiability question of a *specific* tensor is still widely open. In [Linear Algebra Appl. **18** (1977), no. 2, 95–138; MR0444690], J. B. Kruskal provided a well-known criterion for tensor identifiability, which is applicable for small values of rank r. This criterion has been refined in many ways over the years (e.g., in [L. Chiantini, G. M. Ottaviani and N. Vannieuwenhoven, op. cit.]).

The central case which the authors focus on is *ternary septics* (i.e., forms of degree 7 with 3 variables). In this case, Kruskal's original criterion gives answers only for $r\le 6$ and the reshaped Kruskal's method of [L. Chiantini, G. M. Ottaviani and N. Vannieuwenhoven, op. cit.] or the catalecticant method developed in [A. Massarenti, M. Mella and G. Staglianò, J. Symbolic Comput. **87** (2018), 227–237; MR3744347] works only for $r\le 10$. The main result of the paper under review proves that a ternary septic T with $T=L_1^7+\cdots+L_{11}^7$, a minimal decomposition of length 11 (i.e., the only remaining case of the subgeneric ranks in $d=7$, $n=3$), has rank 11 and is identifiable under some maximality conditions on a higher Kruskal's rank of T (which is true for any T outside of a Zariski closed set of measure zero).

The main ingredients for the proof are a detailed analysis of the Hilbert function of the finite points in $\mathbb{P}^2$ and a geometric use of postulations such as the Cayley-Bacharach property of points. As pointed out in Section 6, the main result can be generalized a bit further.

Kangjin Han

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.