MR3819100 14N05 14Q15 15A69
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Identifiability beyond Kruskal's bound for symmetric tensors of degree 4. (English summary)

Atti Accad. Naz. Lincei Rend. Lincei Mat. Appl. **29** (2018), *no. 3,* 465–485.

Kruskal's well-known criterion for identifiability of tensor decompositions applies only within a certain bounded numerical range and applies to general tensor decompositions, ignoring any symmetry of the tensor being decomposed. This paper specializes to (fully) symmetric decompositions of symmetric tensors, or equivalently, Waring decompositions of homogeneous polynomials. The authors specialize further to polynomials (symmetric tensors) of degree 4, i.e., homogeneous quartics. In this case they are able to extend Kruskal's criterion for identifiability to a criterion for symmetric identifiability, valid in a numerical range extending one step further than the original range for Kruskal's criterion.

Let F be a homogeneous quartic in $n+1$ variables, with a decomposition $F=\sum_{i=1}^r \ell_i^4$, the ℓ_i linear forms. Assume that the decomposition is minimal in the sense that F does not lie in the linear span of any $r-1$ terms of the sum; equivalently, the terms ℓ_i^4 are linearly independent. A first question is whether r is minimal, in which case one says that F has Waring rank r, or if there exists some other decomposition with fewer than r terms. When r is minimal (F has Waring rank r), a second question is whether the terms ℓ_i are uniquely determined (up to order and scale), in which case F is called "identifiable".

A version of Kruskal's criterion asserts, in the degree 4 case, that when the points ℓ_i are in linearly general position (LGP) and $r\le 2n$, then F has Waring rank r and is identifiable. (More general statements apply to other degrees, or general tensors.) In this paper, a criterion for identifiability is given for quartics with rank $r\le 2n+1$. When $r\le 2n$ it is the same criterion as before. In the new case $r=2n+1$, one checks whether the points $\ell_1,\dots,\ell_{2n+1}$ lie on a rational normal curve. This can be computed effectively, and indeed the authors have implemented their test in the Macaulay2 computer algebra system.

The result is a very appealing application of classical algebraic geometric techniques to a problem of current interest in tensor decompositions. The authors recall and extend a classical result of Castelnuovo on rational normal curves passing through a finite set of points; and they recall and extend a result of Geramita, Kreuzer, and Robbiano on Hilbert functions of finite sets of points with the Cayley-Bacharach property. The paper is self-contained as it includes good reviews of needed background from algebraic geometry and from tensor decompositions, so readers from either of these areas will be able to learn what they need from the other side. *Zach Teitler*

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