A truncated trapezoidal approach for the fractional power of operators

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Abstract

In this talk we present numerical approximations of $\mathcal{L}^{-\alpha}, \alpha \in (0,1)$. Here $\mathcal{L}$ is a self-adjoint positive operator acting in an Hilbert space $\mathcal{H}$, with positive real spectrum.

This problem finds immediate application when solving equations involving a fractional diffusion term like $-(-\Delta)^{\alpha}$, where Δ denotes the standard Laplacian.

Starting from the integral representation

$$\mathcal{L}^{-\alpha} = \frac{2\sin(\alpha\pi)}{\pi} \int_0^{+\infty} t^{2\alpha-1}(\mathcal{I} + t^2\mathcal{L})^{-1} dt, \qquad \alpha \in (0,1),$$

where $\mathcal{I}$ is the identity operator in $\mathcal{H}$, after suitable changes of variables we consider approximations based on truncated trapezoidal rules. We show that the error decays exponentially. The proposed approach is therefore very fast and the error estimates obtained allow to select *a-priori* the number of quadrature points necessary to obtain a given accuracy.

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