The discrete line integral: a simple tool to devise energy-preserving Runge–Kutta methods

Luigi Brugnano [*] Felice Iavernaro [†]

Workshop "Donato Trigiante: il matematico, l'uomo, le idee"

Abstract

In this presentation, we retrace the path of investigation leading up to the discovery of Runge–Kutta energy-preserving methods. These integrators, when applied to canonical Hamiltonian systems, yield a precise conservation of the Hamiltonian function, that is the total energy of the system. This specific topic has been deeply investigated during the past decades, and the first unsuccessful attempts culminated in the wrong general feeling that such methods could not even exist.

Donato's methodological principle of *focussing on the problem rather than on the method* has proved crucial in this context. The new formulae naturally emerged after a trivial discretization of the *line integral* associated with the vector field of the dynamical system and, surprisingly, this approach unveiled a wide class of, until then unknown, Runge–Kutta integrators. The simplicity of the tool and the techniques employed reflect Donato's teachings that *ideas are much stronger than formalism*, and form the master route to advance the level of knowledge.

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[*]Università di Firenze
[†]Università di Bari