Tridiagonal matrices and fast and accurate computation of symmetric Gaussian quadrature rules

Teresa Laudadio * Nicola Mastronardi * Paul Van Dooren †

Workshop "Donato Trigiante: il matematico, l'uomo, le idee"

Abstract

Given the integral

$$\int_{-a}^{a} f(x)\omega(x)dx,$$

with f a continuous function and ω a positive weight, the Golub–Welsch algorithm [1] is the classical way to compute the knots x_i and the weights $w_i,\ i = 1, \ldots, n$, of the Gaussian quadrature rule

$$\sum_{i=1}^{n} f(x_i)w_i.$$

In particular, the knots x_i are the eigenvalues of a tridiagonal matrix of order n, called *Jacobi* matrix, whose nonzero entries are the coefficients of the three–term recurrence relation of the sequence of orthogonal polynomials associated to ω. Moreover, known $x_i,\ i = 1, \ldots, n$, the corresponding weight w_i can be obtained from the first component of the eigenvector associated to x_i.

If ω is a symmetric function, the knots x_i and the weights $w_i,\ i = 1, \ldots, n$, can be obtained by solving a tridiagonal eigenvalue problem of size $n/2$ [2].

Exploiting the algorithm proposed in [2], we derive an efficient and highly accurate method to compute the knots and the weights of Gaussian quadrature rules corresponding to symmetric weights ω.

References

1. G.H. Golub, J.H. Welsch. Calculation of Gauss quadrature rules. *Math. Comp.* 23 (1969) 221–230.

2. G. Meurant, A. Sommariva. Fast variants of the Golub and Welsch algorithm for symmetric weight functions in matlab. *Numer. Algor.* 67 (2014) 491–506.

*IAC-CNR, Bari

†Catholic University of Louvain