On Lubich's convolution quadratures and their application to space-time boundary integral equations

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Abstract

We first recall space-time boundary integral representations of some exterior wave propagation problems. These include, in particular, the Dirichlet problem for the classical wave equation defined over the entire exterior of a single bounded domain or a bounded multidomain region. Then, we briefly describe a numerical approach to solve these integral equations, obtained by coupling a second order (time) discrete convolution quadrature, first proposed and examined by Ch. Lubich in 1988, with classical Galerkin and collocation (space) boundary element methods, defined by a continuous piecewise linear approximant.

While for the above mentioned Dirichlet problem, stability and convergence of the discrete convolution - Galerkin numerical scheme have been proved, for the corresponding collocation approach we only have numerical evidence that the same properties should also hold. To prove that this conjecture is indeed true, as a first step, we show that for the wave equation Dirichlet problem the collocation approach is formally equivalent to a discretization of the Galerkin one, the latter being obtained by approximating its inner product integral by a proper quadrature rule.

To investigate this relationship between the two methods, in the case of the 2D problem we have performed a deeper study of the behavior of the coefficients of the Lubich quadrature. This in turn allowed us to accurately estimate the behavior of the elements of the linear system defined by the Galerkin approach, and also to derive bounds for the errors produced by the above Galerkin inner product discretization. Some results have then been obtained for behavior of the associated collocation system.

References

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