Matrix-Oriented Discretization Methods for Evolutionary Problems

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Abstract

An interesting class of PDE evolutionary problems is given by reaction-diffusion systems where the coupling between diffusion and nonlinear kinetics can lead to the so-called Turing instability. In this case, a variety of spatial patterns can be attained as stationary solutions for longtime integration. To capture the morphological peculiarities of the Turing patterns, a very fine space discretization may be required, limiting the use of standard (vector-based) ODE solvers in time because of excessive computational costs.

We show that the structure of the diffusion matrix can be exploited to build matrix-based versions of some classical time integrators, such as Implicit-Explicit (IMEX) schemes. In particular, we consider finite differences on square domains and classical Lagrangian FEM of order $k = 1, .., 4$ on x-normal domains and even on special surfaces. In the first case, the discrete problem is then reformulated as a sequence of Sylvester matrix equations, that we solve by the *reduced approach* in the associated spectral space [1,3]. On general domains, at each time step, *multiterm Sylvester matrix equations* must be solved, where the additional terms account for the geometric contribution of the domain shape. In this case, we solve the matrix equations by the matrix-oriented form of the Preconditioned Conjugate Gradient (MO-PCG) method [2].

We illustrate our findings by applying the IMEX Euler method for: i) the semilinear heat equation and ii) the approximation of Turing patterns in the reaction-diffusion DIB model describing metal growth during battery charging processes. Encouraging results justify the matrix approach in terms of execution times and memory storage.

References

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