Chapter 6

Notes and References

The approach of using discrete line integrals has been used, at first, by Iavernaro and Trigiante, in connection with the study of the properties of the trapezoidal rule [26, 27, 28].

It has been then extended by Iavernaro and Pace [24], thus providing the first example of conservative methods, basically an extension of the trapezoidal rule, named *s-stage trapezoidal methods*: this is a family of energy-preserving methods of order 2, able to preserve polynomial Hamiltonian functions of arbitrarily high degree.

Later generalizations allowed Iavernaro and Pace [25], and then Iavernaro and Trigiante [29], to derive energy preserving methods of higher order.

The general approach, involving the shifted Legendre polynomial basis, which has allowed a full complete analysis of HBVMs, has been introduced in [6] (see also [5]) and, subsequently, developed in [7].

The Runge-Kutta formulation of HBVMs, along with their connections with collocation methods, has been studied in [9].

The isospectral property of HBVMs has been also studied in [8], where the *blended* implementation of the methods has been also introduced.

Computational aspects, concerning both the computational cost and the efficient numerical implementation of HBVMs, have been studied in [3] and [8].

Relevant examples have been collected in [4], where the potentialities of HBVMs are clearly outlined, also demonstrating their effectiveness with respect to standard symmetric and symplectic methods.

Blended implicit methods have been studied in a series of papers [2, 10, 11, 12, 13, 14, 15, 16, 30] and have been implemented in the two computational codes `BiM` and `BiMD` [32].

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