Exponentially fitted methods applied to fourth order boundary value problems

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We consider the numerical solution of the following fourth-order boundary value problem:

\[ y^{(4)} + f(t) y = g(t), \quad a \leq t \leq b \]

subject to the boundary conditions

\[ y(a) = A_1, \quad y''(a) = A_2, \quad y(b) = B_1, \quad y''(b) = B_2. \]

We solve this problem by transforming the differential problem into a set of five-point finite difference equations whose coefficients are determined by a couple \((K, P)\), where \(K + 1\) indicates the number of conditions related to polynomials and \(P + 1\) indicates the number of conditions related to exponentials (or trigonometric functions). These methods, which we call exponentially fitted, contain a parameter which can be tuned for the problem at hand. Starting from the expression for the error, we will discuss ways to choose this parameter.

This talk is related to the talk of Davy Hollevoet, who discusses the same technique for second order boundary value problems.