A time-adaptive integrator based on Radau methods for Advection Diffusion Reaction PDEs

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The numerical integration of time-dependent PDEs, especially of Advection Diffusion Reaction type, for two and three spatial variables (in short, 2D and 3D problems) in the MoL framework is considered. The spatial discretization is made by using Finite Differences and the time integration is carried out by means of the *L-stable, third order formula* known as the two stage Radau IIA method. The main point for the solution of the large dimensional ODEs is not to solve the stage values of the Radau method until convergence (because the convergence is very slow on the stiff components), but only giving a very few iterations and take as advancing solution the latter stage value computed. The iterations are carried out by using the Approximate Matrix Factorization (AMF) coupled to a Newton-type iteration as indicated in [2], which turns out in an acceptably cheap iteration, since it reminds of the Alternating Directions Methods (ADI) of Paeceman and Rachford (1955). We also discuss some variants of the (AMF) presented in [2], just by modifying a parameter in the pre-conditioner used in the iteration, which allows to win some stability in the advancing Radau formula, especially for 3D-problems. The stability regions of the new approach are compared with those ones in [1], which consider the stability of a few Linear Multistep and DIRK methods. On the other hand, variable time-step integrations are considered by incorporating a local error estimate with convenient stability properties on the stiff components, which makes the overall integrator much more efficient. Numerical results on a few real-life problems are presented and some conclusions about our method and other standard solvers such as, VODPK and IMEXRKC, are drawn.

References

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