A new approach based on Piecewise Linear Systems for the numerical solution of obstacle problems

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Abstract

Piecewise Linear Systems are linear systems whose coefficient matrix is a piecewise constant function of the solution itself. Such systems have been recently introduced in the literature [1, 2], where their application to the numerical solution of flows in porous media is also presented. In this talk we will consider their application to the numerical solution of obstacle and parabolic obstacle problems which are discretized by a standard finite difference scheme [3]. A semi–iterative Newton–type method is used for solving the specific piecewise linear systems associated with such kind of problems and the monotone convergence of such procedure is proved, implying finite convergence. Numerical examples show that typically a few iterates are necessary to obtain the exact solution of the considered piecewise linear systems. As a consequence, we obtain a very efficient method for the numerical solution of obstacle and parabolic obstacle problems which are very important in real–life applications.

References

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[3] L. Brugnano, A. Sestini. Iterative Solution of Piecewise Linear Systems for the Numerical Solutions of Obstacle Problems, (2009) submitted.