

# Solving the beam bending problem with an unilateral Winkler foundation

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## Abstract

Our work is going to deal with the bending of a beam resting on an unilateral elastic foundation and develops further the ideas from the article [?]. In some cases the beam has fixed connection with the foundation. Such problems are linear. However there are applications where the beam is not connected with the foundation. This so-called unilateral case represents an interesting nonlinear problem and cannot be solved by easy means. We propose here first a new formulation of this problem which is based upon the idea of a decomposition. This way we can convert the usual variational formulation of our problem to a saddle-point formulation.

In the second part of this paper we will deal with a numerical solution using the finite element method. The system of equations for the saddle point is nonlinear and nondifferentiable. It can be handled by the transformation to a complementarity problem which is solved by the nonsmooth Newton method.

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