Active and passive symmetrization of Runge-Kutta Lobatto IIIA methods

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Abstract. Symmetric methods such as those of the Gauss and Lobatto IIIA families of high order are of special interest because their numerical solutions possess asymptotic error expansions in even powers of the stepsize. This property can therefore be exploited by acceleration techniques such as extrapolation to increase the order by two at a time. Although symmetric methods can have high classical order and can be A-stable, their order is often observed to be lower when applied to stiff problems. This order reduction phenomenon will weaken the advantage of performing extrapolation because of the uncertainty in choosing the correct extrapolation formula. In addition to this difficulty, the weak damping property of symmetric methods may cause extrapolation of symmetric methods to be inefficient. To overcome these issues, Chan [1] generalized the concept of *smoothing* first introduced by Gragg [5]. The process called *symmetrization* is carried out by a *symmetrizer* which is constructed by taking the composition of two symmetric Runge-Kutta methods but with different weights [1]. The weights are chosen to preserve the h^2-asymptotic error expansion and to provide damping, thus generalizing the smoothing formula used by Dahlquist and Lindberg [3] for the implicit midpoint and trapezoidal rules.

For a given symmetric method of order $p \geq 4$, a symmetrizer is constructed to satisfy the order conditions to as high an order as possible and to achieve damping for stiff problems. In the case of an s-stage Gauss method with nonsingular A, the weight vector u has s components and these can be used to satisfy the damping condition $\widetilde{R}(\infty) = 0$ and conditions for order $2s-1$. Gauss methods with symmetrization have been shown to be robust in solving stiff linear and nonlinear problems [2, 4]. However, in the case of the Lobatto IIIA methods where A is singular, the weight vector u has $s-1$ components. The study of stiff order behaviour for the Prothero-Robinson problem suggests that the symmetrized 3-stage and the symmetrized 4-stage Lobatto IIIA methods exhibit order-4 and order-6 behaviour respectively. In this paper we present numerical results in the constant stepsize setting that show that symmetrization is more efficient when performed with extrapolation for mildly stiff and stiff linear and nonlinear problems. We also present results that show the most efficient strategy is to combine passive symmmetrization with passive extrapolation.

Keywords: Lobatto IIIA methods, stiff problems, symmetrization, active, passive, extrapolation
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REFERENCES

1. R. P. K. Chan, *Generalized symmetric Runge-Kutta methods*, Computing. 50 (1993) 33–49.
2. R. P. K. Chan and A. Gorgey, *Active and passive symmetrization of Runge-Kutta Gauss methods*, Article in press (http://dx.doi.org/10.1016/j.apnum.2011.06.013).
3. G. Dahlquist and B. Lindberg, *On some implicit one-step methods for stiff differential equations*, Dept. of Inf. Proc., Roy. Inst. Tech., Stockholm, Report TRITA-NA-7302, 1973.
4. A. Gorgey, *Extrapolation of symmetrized Runge-Kutta methods*, Ph.D Thesis, University of Auckland, 2012.
5. W.B. Gragg, *On extrapolation algorithm for ordinary initial value problems*, SIAM J. Numer. Anal. 2 (1965) 384–403.