Generalized Adams Methods for Fractional Differential Equations

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We consider the application of convolution quadratures for the numerical solution of Initial Value Problems for Fractional Differential Equations (FDEs). Methods of this type currently available in the literature are, for example, Adams product quadrature rules or Fractional Linear Multistep Methods, [2, 3]. It is known that these schemes suffer of the usual order barrier for A-stable methods. In particular, in [3] it was proved that the order of an A-stable convolution quadrature cannot exceed two. Clearly, this result represents an extension of the famous second Dahlquist barrier for linear multistep methods (LMMs) for ordinary differential equations. This barrier can be overcomed if LMMs are used as Boundary Value Methods (BVMs) that is if the discrete problem generated by a k-step LMM is completed by imposing k_1 and $k_2 = k - k_1$ boundary conditions, [1]. By virtue of this result, we investigate here if the BVM approach is successful in overcoming the barrier established in [3] for convolution quadrature methods. In particular, we introduce a generalized version of implicit Adams product quadrature rules that we call *Fractional Generalized Adams methods* (FGAMs) and we study their accuracy and stability properties. The boundary loci reported show that, when used as Boundary Value Methods, these schemes are always A-stable.

References

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