

Calculations of the Morphology Dependent Resonances

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Abstract

The aim of this talk is to present the progress in the numerical simulation of the so called ‘whispering gallery’ modes (WGM) occurring inside a prolate spheroidal cavity. These modes are strongly confined within the spheroid structure along the equatorial line and have an extremely high-quality factor Q .

The notion ‘whispering gallery’ was introduced by John William Strutt (Lord Rayleigh) in context of sound waves observed in the Whispering Gallery of the St Paul’s Cathedral in London, where a whisper against its wall at any point is audible to a listener with an ear held to the wall at any other point of the gallery. The WGM phenomenon arises in many branches of physics, like acoustics or electronics, and finds numerous applications there.

Since the related eigenvalue problem is separable in spheroidal coordinates, we deal with two ODEs, related to the angular and radial coordinates. Though separated, the equations are coupled via the equation coefficients and take the form of a singular, two-parameter Sturm–Liouville problem. Solution of such problems requires special care. The difficulty is caused by the high localization of WGMs, since the related ODE solutions strongly vary in very narrow intervals, and decay exponentially fast outside these intervals. Following the idea proposed in [1] for the solution of a singular, self-adjoint Sturm–Liouville problem, we combine the Prüfer angle technique, applied to provide the starting guess for the eigenvalues, with a high order finite difference scheme for the accurate calculation of the associated eigenfunctions and improved eigenvalues. We point out that the technique presented here is also applicable for any other separable geometry.

We illustrate the approach by numerical simulations showing highly localized WGMs inside a spheroid.

References

- [1] P. Amodio, T. Levitina, G. Settanni, and E.B. Weinmüller, *On the calculation of the finite Hankel transform eigenfunctions*, J. Appl. Math. Comput. in press, DOI 10.1007/s12190-013-0657-1

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