Analytical and numerical treatment of singular linear BVPs with unsmooth inhomogeneity

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We investigate analytical and numerical properties of systems of linear ordinary differential equations with unsmooth nonintegrable inhomogeneities and a time singularity of the first kind. Boundary value problems of our interest have the form:

$$y'(t) = \frac{M}{t}y(t) + \frac{f(t)}{t},\ t \in (0,1], \quad B_0y(0) + B_1y(1) = \beta,$$

where $f \in C[0,1]$, $M \in \mathbb{R}^{n\times n}, B_0, B_1 \in \mathbb{R}^{m\times n}$ and $\beta \in \mathbb{R}^m$. We are especially interested in specifying the structure of general linear two-point boundary conditions guarantying existence and uniqueness of solutions which are continuous on a closed interval including the singular point $t = 0$. It turns out that constant matrices B_0 and B_1 are subject to certain restrictions for a problem to be well–posed. Moreover, we study the convergence behaviour of collocation schemes applied to solve the problem numerically. Our theoretical results are supported by numerical experiments.