Fixed point problem associated with state-dependent impulsive boundary value problems

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Abstract. We investigate a fixed point problem in the Sobolev space $\left(\mathbb{W}^{1,\infty}([a,b];\mathbb{R}^n)\right)^{p+1}$ which is connected to the boundary value problem

$$z'(t) = f(t,z(t)), \quad \text{a.e. } t \in [a,b] \subset \mathbb{R}, \quad \ell(z) = c_0,$$

with the state-dependent impulses

$$z(t+) - z(t-) = J_i(t,z(t-)), \quad i = 1,\dots,p,$$

where the impulse instants $t \in (a,b)$ are determined as solutions of the equations

$$t = \gamma_i(z(t-)), \quad i = 1,\dots,p.$$

We assume that $n,p \in \mathbb{N}$, $c_0 \in \mathbb{R}^n$, the vector function f satisfies the Carathéodory conditions on $[a,b]\times\mathbb{R}^n$, the impulse functions J_i, $i=1,\dots,p$, are continuous on $[a,b]\times\mathbb{R}^n$, and the barrier functions γ_i, $i=1,\dots,p$, are continuous on $\mathbb{R}^n$. The operator ℓ is an arbitrary linear and bounded operator on the space of left-continuous regulated (i.e. having finite one-sided limits at each point) on $[a,b]$ vector valued functions and is represented by the Kurzweil-Stieltjes integral. Provided the data functions f and J_i are bounded, transversality conditions which guarantee that this fixed point problem is solvable are presented. As a result it is possible to realize a construction of a solution of the above impulsive problem.

Keywords: System of ODEs of the first order, state-dependent impulses, general linear boundary conditions, transversality conditions, fixed point problem
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