Performance of “Look-Ahead” Linear Multistep Methods
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Abstract
We are concerned with the initial-value problem of ordinary differential equations (ODEs):
\[ \frac{dy}{dx} = f(x, y) \quad (a \leq x \leq b), \quad y(a) = y_I. \]
LALMM, which stands for “look-ahead” linear multistep methods, is a new class among the discrete variable methods (DVMs) for the problem. Its mechanism is as follows: Assume that we look for the numerical solution of the \((n + k)\)-th step-point when the back-values \(y_n, y_{n+1}, \ldots, y_{n+k-1}\) and a preassigned initial guess \(y_{n+k}^0\) are available. First, we look ahead for the \((n + k + 1)\)-st step-point by
\[
y_{n+k+1}^0 + \alpha_k y_{n+k}^0 + \sum_{i=0}^{k-1} \alpha_i y_{n+i} = h \left( \beta_k f(x_{n+k}, y_{n+k}^0) + \sum_{i=0}^{k-1} \beta_i f(x_{n+i}, y_{n+i}) \right),
\]
which can be regarded as a predictor. Then, correct the look-for value by
\[
y_{n+k}^1 + \sum_{i=0}^{k-1} \alpha_i^* y_{n+i} = h \left( \beta_{k+1}^* f(x_{n+k+1}, y_{n+k+1}^0) + \beta_k^* f(x_{n+k}, y_{n+k}^0) + \sum_{i=0}^{k-1} \beta_i^* f(x_{n+i}, y_{n+i}) \right).
\]
When a (local) convergence attains, i.e., the estimation \(\|y_{n+k}^1 - y_{n+k}^0\| \leq \delta_{TOL}\) holds for a pre-assigned error tolerance \(\delta_{TOL}\), we complete the current step and advance to the next step. Otherwise, we replace \(y_{n+k}^0\) by \(y_{n+k}^1\) and iterate prediction and correction. Note that we employ equi-distant step points \(\{x_n\}\) and approximations \(\{y_n\}\) on them.

The core issue of numerical analysis of new methods is whether they can perform better than the existing methods. We derived several LALMM schemes of two-step family (LALTM) and examine their performance through test examples of ODEs. We will report the test results of LALTMs by several numerical examples and describe a possible way to overcome their difficulties shown in the examples.

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