Wavelet Discretization of Black-Scholes Equation

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Abstract.

Option products are frequently traded in the financial markets. To price these options, advanced mathematical models are employed, yielding multidimensional parabolic partial differential equations of the convection-diffusion type. And it is necessary for financial institutions to have at one's disposal efficient, stable, and robust numerical methods to compute their fair values. We start here with the one-dimensional Black-Scholes equation for pricing European options [1]. We use an operator splitting method to split the arising system of equations into an symmetric part and into an unsymmetric part. Then, we employ the $\theta-$scheme for the time discretization and wavelets for the space discretization.

In this contribution, we use the wavelet basis based on Hermite cubic splines proposed in [2] based on ideas from [3, 4] with optimized condition numbers. The main advantage of this wavelet basis consists in the fact that stiffness matrices corresponding to the wavelet discretization of the Black-Scholes operator are sparse. Then, a matrix-vector multiplication can be performed exactly with linear complexity.

Using wavelet methods, the continuous problem is transformed into a well-conditioned discrete problem. And once a non-symmetric problem is given, squaring yields a symmetric positive definite formulation. However squaring usually makes the condition number of discrete problems substantially worse. We show here that in wavelet coordinates a symmetric part of the discretized equation dominates over an unsymmetric part in the standard economic environment with low interest rates. It provides some justification for using a fractional step method with implicit treatment of the symmetric part of the weak form of the Black-Scholes operator and with explicit treatment of its unsymmetric part. Consequently, the arising system of equations can be efficiently preconditioned using an wavelet based preconditioning. Numerical examples are given.

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