Functional Continuous Runge—Kutta Methods for Cross-dependent Retarded Sytems

Alexey S. Eremin

Abstract

This talk generalises the results presented at the Symposium last year. It deals with numerical methods for systems of retarded functional differential equations (RFDEs) of two equations in which the right-hand sides are cross-dependent of the retarded values of the unknown functions, i.e.

$$\begin{cases} \dot{u}(t) = f(t, u(t), v_t), \\ \dot{v}(t) = f(t, v(t), u_t), \end{cases}$$

where u_t and v_t mean the delayed u and v functions. Such systems cover larger class of problems and getting an advantage over the known methods is our goal.

Special explicit methods of Runge—Kutta type for solving RFDEs were suggested by Tavernini already in the 1970's. He constructed several methods up to order four. Only in recent years these methods have been further developed and generalised into a large class of methods named Functional Continuous Runge—Kutta methods (FCRKs) for RFDEs. FCRKs were reviewed in Acta Numerica in 2009 (along with other approaches to delay differential equations and RFDEs solution). Order conditions and examples of methods were presented there.

In early 1990's Olemskoy developed methods exploiting special structure of ordinary differential equations systems. They were inspired by Runge—Kutta—Nyström methods for second order equations, which have much better stage to order ratios than classic Runge—Kutta methods. He showed that if some right-hand sides in the system are independent of the function, to which derivative they correspond, and some other functions, it is possible to construct Runge—Kutta type methods with fewer stages for all (or at least some) of the system equations. These methods are a generalisation of Runge—Kutta methods, since classic RKs and as well Runge—Kutta—Nyström can be obtained as their reduction.

The methods analogous to Runge—Kutta—Nyström methods can be developed for the RFDE problems of the form $\ddot{u}(t) = f(t, u_t, \dot{u}(t))$. They have the same order of convergence with less stages than the FCRKs for the corresponding first order equations, and are thus more efficient. We look at a larger class of problems, which includes the mentioned second order equation as a particular case.

In the current work the construction of FCRKs for the systems of RFDEs analogous to the systems considered by Olemskoy in terms of special structure is discussed. It is shown that for the same convergence order (discrete and/or uniform) methods can be constructed with fewer stages than FCRKs for the general RFDEs systems. Order conditions and example methods are presented. Test problems are solved, demonstrating the declared convergence order of the new methods.