Numerical methods for multi scale hyperbolic problems, with application to multi fluid and sedimentation.

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In this project we consider the development of a semi-implicit finite volume scheme for the numerical solution of multi scale hyperbolic systems. We consider two model problems: multi-fluid compressible Euler equations [1] and Shallow water equations equipped with a bedload sediment transport equation [4]. In both cases the equations are discretized on a non-staggered grid, which is a generalization of the method proposed in [2]. For the multi fluid problem, in order to avoid oscillations generated at material interfaces, we make use of the so-called ghost fluid method which utilizes a level set function in order to locate the material interface and then a simple extrapolation of the EOS for the computation of the splitted solution using the semi-implicit scheme. On the shallow water problem we develop a semi-implicit scheme in which surface water waves are treated implicitly, while the slow wave corresponding to sediment evolution is treated explicitly. Second order in space is obtained by WENO reconstruction for all the upwind derivatives and classical three point central scheme for the rest. Second order in time is achieved through an implicit-explicit Runge-Kutta scheme [3]. Finally, the CFL stability condition only depends at the fluid velocity which means that only the material CFL must be less than one. Several numerical tests are performed, which show the efficiency of the approach, when fast waves have a negligible amplitude.

References

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