Numerical solution of fractional integro-differential equations with non-local boundary conditions

Arvet Pedas, Enn Tamme, Mikk Vikerpuur
University of Tartu, Estonia

We consider a class of fractional integro-differential equations with non-local boundary conditions:

$$(D_*^\alpha y)(t) + h(t)y(t) + \int_0^t K(t,s)y(s)ds = f(t),\ 0 \le t \le b,$$

$$\gamma_0 y(0) + \gamma_1 y(b_1) + \gamma_2 \int_0^{b_2} y(s)ds + \gamma_3 \int_0^{b_3} (D_*^\beta y)(s)ds = \gamma,$$

where $b > 0$, $b_1, b_2, b_3 \in (0, b]$, $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma \in \mathbb{R} := (-\infty, \infty)$ and D_*^α is the Caputo differential operator of order α. We assume that $0 < \beta \le \alpha < 1$, $h, f \in C[0, b]$, $K \in C(\Delta_b)$, $\Delta_b = \{(t, s) : 0 \le s \le t \le b\}$ and $\gamma_0 + \gamma_1 + \gamma_2 b_2 \ne 0$.

The problem is reformulated as a Volterra integral equation of the second kind with respect to the fractional derivative of the solution of the original problem. First, the smoothness of the exact solution is studied. On the basis of the obtained regularity properties, by using spline collocation techniques, an efficient method for the numerical solution of the problem is proposed. The convergence of the proposed algorithms is shown and a global super-convergence result is presented. A numerical illustration is also given.