Numerical solution of fractional integro-differential equations with weakly singular kernels

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We consider a class of fractional weakly singular integro-differential equations with nonlocal boundary conditions:

$$(D_*^\alpha y)(t) + h(t)y(t) + \int_0^t (t-s)^{-\kappa} K(t,s)y(s)ds + \int_0^t (t-s)^{-\kappa_1} K_1(t,s)(D_*^\beta y)(s)ds = f(t),$$

$$\gamma_0 y(0) + \gamma_1 y(b_1) + \gamma_2 \int_0^{b_2} y(s)ds = \gamma.$$

Here $0 \le t \le b$, $b > 0$, $b_1, b_2 \in (0,b]$, $\gamma_0, \gamma_1, \gamma_2, \gamma \in \mathbb{R} := (-\infty, \infty)$ and D_*^α, D_*^β are the Caputo differential operators of orders α, $\beta \in (0,1)$. More precisely, we assume that $0 < \beta < \alpha < 1$, $\kappa, \kappa_1 \in [0,1)$, $h, f \in C[0,b]$, $K, K_1 \in C(\Delta_b)$, $\Delta_b = \{(t,s) : 0 \le s \le t \le b\}$ and $\gamma_0 + \gamma_1 + \gamma_2 b_2 \ne 0$.

By using an integral equation reformulation, we first study the existence, uniqueness and smoothness of the exact solution. On the basis of this information and spline collocation techniques, the numerical solution of the problem is discussed. The convergence of the proposed algorithms is shown and global error estimates are derived. A numerical illustration is also given.