Some issues with numerical treatment of delay differential equations

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Abstract

In the talk some issues which most commonly arise in the numerical solution of discrete delay differential equations (DDEs),

$$\begin{cases} \dot{u}(t) = f(t, u(t), u(t-\tau)), & t \geq t_0, \\ u(t) = \phi(t), & t \leq t_0, \end{cases}$$

and neutral delay differential equations (NDDEs)

$$\begin{cases} \dot{u}(t) = f(t, u(t), u(t-\tau), \dot{u}(t-\sigma)), & t \geq t_0, \\ u(t) = \phi(t), & t \leq t_0, \end{cases}$$

where the delays τ and σ can be functions of t and/or $u(t)$ (time- and state-dependent delays), are considered.

At first, we consider *overlapping*, i.e. the situation when the delay is smaller than the ste-size and explicit methods (if not specially designed) become implemented fully implicitly. We present explicit Runge–Kutta methods for overlapping.

Next, we study error estimation techniques and how it is different from ordinary differential equations case. Embedded methods are compared to residual control.

One of the most important difficulties are discontinuities of solution derivatives and necessity to locate discontinuity points to preserve the accuracy and convergence order of the method. Several approaches are compared.

Finally, we mention events location, i.e. locating points where some conditions are satisfied.

We also present numerical results comparing a method, where all the above-mentioned issues are effectively solved, with the explicit Matlab solver ddesd.