Stability Analysis of Numerical Methods Using a Linear Test SDE with Delay and Non-delay in a Diffusion Term

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We are concerned with stabilized numerical methods for the strong approximation to the solution of stochastic delay differential equations (SDDEs). A class of such methods is the class of the stochastic theta methods. Comparing with numerical methods for stochastic differential equations without delay, the issues to analyse numerical methods for SDDEs are much more complicated. Nevertheless, Huang, Gan, and Wang [1] have analysed mean square (MS) stability properties of the stochastic theta methods when the methods are applied to the test equation

$$\begin{aligned} &\mathrm{d}y(t) = \lambda y(t)\mathrm{d}t + (\sigma_1 y(t) + \sigma_2 y(t-\tau))\mathrm{d}W(t), \qquad t \geq 0, \\ &y(0) = \Psi(t), \quad t \in [-\tau, 0], \end{aligned}$$

where $\lambda < 0$ and $\sigma_1, \sigma_2 \in \mathbb{R}$, $\tau > 0$ is a constant delay, $W(t)$ is a scalar Wiener process, and Ψ is continuous on $[-\tau, 0]$. In the analysis, they have proposed a theorem for the stochastic theta methods.

Incidentally, Komori, Eremin and Burrage [2] have studied stabilized explicit numerical methods for SDDEs and have successfully derived stochastic orthogonal Runge–Kutta–Chebyshev (SROCK) methods. Using a simple scalar test equation correponding to the above test equation when $\sigma_1 = 0$, they have investigated stability properties of the SROCK methods. In the present talk, we extend a theorem proposed by Huang et al. [1] to a proposition in a general form. After that, we will apply it to the SROCK methods and the explicit Euler–Maruyama (EM) method in order to analyse their MS stability properties.

REFERENCES

[1] C. Huang, S. Gan, and D. Wang, J. Comput. Appl. Math. **236**, 3514–3527 (2012).
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