Finite Element Methods with the Special Graded Mesh for Elliptic Equation with Degeneration on the Entire Boundary

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For boundary value problems with a singularity at the boundary of the domain the rate of convergence of the approximate solution by the finite element method and the finite difference method on quasi-uniform grids is insufficient. For such problems we defined an R-generalized solution [1,2] and we constructed and investigated the weighting finite element method for finding an approximate solution [3–5]. The main advantage of this method is the constant rate of convergence O(h) regardless of the size and magnitude of the singularity. The complexity of the realization of the weight finite element method consists in the selection of optimal parameters at which the rate of convergence of the approximate solution to the exact solution is close to the theoretical one.

In this paper we consider the Dirichlet problem for an elliptic equation with degeneration of the solution on entire boundary of the two-dimensional domain. In [7] for this problem a finite element method was constructed and the convergence of this method was established. The paper [6] singles out the weighted subspace of functions for which the approximate solution converges to an exact solution with a speed O (h) on mesh with the special compression of nodes to the boundary (see [7]). The compression parameters depend on the constructed subspace. Here we carried out a comparative numerical analysis of finite element methods on quasi-uniform meshes and meshes with the special compression of nodes to the boundary. We obtained experimental confirmation of theoretical estimates and demonstrated the advantage of the proposed method over the classical finite element method. By analogy with [5], we found that it is impossible to use FEM with a strong thickening of mesh and introduction of an R-generalized solution is required. The existence and uniqueness of the R-generalized solution for this problem was proved in [8].

References

[1] Rukavishnikov V A 1989 Dokl. Akad. Nauk SSSR **309** 1318-1320

[2] Rukavishnikov V A Doklady Mathematics 2014 **90** 562–564

[3] Rukavishnikov V A and Rukavishnikova H I 2010 J. Comput. Appl. Math. **234** 2870-2882

[4] Rukavishnikov V A and Mosolapov A O 2012 J. Comp. Phys. **231** 2438–2448

[5] Rukavishnikov V A and Rukavishnikov AV 2018 J. Comput. Appl. Math **341** 144–156

[6] Rukavishnikov V A and Rukavishnikova E I 2014 Differential Equations **50** 345–351

[7] Rukavishnikova E I 2011 Information Science and Control Systems **30** 57–64

[8] Rukavishnikov VA and Rukavishnikova E I 2016 Differential Equations **52** 681–685