Convergence of the collocation method for nonlinear singular BVPs with nonsmooth data

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Abstract: We discuss numerical solution of boundary value problems for systems of nonlinear ordinary differential equations with a time singularity,

$$x'(t) = \frac{M(t)}{t}x(t) + \frac{f(t, x(t))}{t}, \quad t \in (0, 1], \quad b(x(0), x(1)) = 0, \tag{1}$$

where M and f are continuous matrix-valued and vector-valued functions, respectively, and b is a continuous nonlinear mapping which is specified according to the spectrum of the matrix $M(0)$. For the case that all eigenvalues of $M(0)$ are not equal zero, we prove new convergence results for the collocation method applied to approximate (1).

A strong motivation for investigating problem class (1) are regular BVPs posed on the semi-infinite interval $[0, \infty)$ and taking the form

$$x'(s) = A(s)x(s) + g(s, x(s)), \quad s \in [0, \infty), \quad b(x(0), x(\infty)) = 0, \tag{2}$$

where s is often identified with time. To solve the problem numerically, we follow the idea to reduce the interval $s \in [0, \infty)$ to the finite domain $t \in [0, 1]$. The advantage of working on a finite small interval [0, 1] while discretizing the analytical problem is evident, however with this transformation, we usually introduce a singularity at $t = 0$ and the problem data become nonsmooth. For the variable transformation $t = e^{-s}$, the problem (1) takes the form (2) and this means that the singularity is of first kind.

Naturally, the way how the singularity affects the proposed numerical method has to be studied and the convergence properties of the numerical scheme (polynomial collocation) need to be established.

As a concrete application we consider the BVP (1) as it arises in the modelling of snow avalanche run-up and run-out. In the leading-edge model describing the dynamics of dry-flowing avalanches five forces are combined to give the total force governing the avalanche's dynamics.

The theory is illustrated by numerical experiments including the simulation of the avalanche run-up.

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