Semi-implicit multivalue almost collocation methods

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We present a family of numerical methods for the solution of ordinary differential equations:

$$\begin{cases} y'(t) = f(y(t)), \quad t \in [t_0, T], \\ y(t_0) = y_0, \end{cases} \tag{1}$$

with $f : \mathbb{R}^k \to \mathbb{R}^k$. Multivalue collocation methods have uniform order of convergence, together with high stability properties, so they are suitable for the solution of stiff problems, which are very common in mathematical models. On the uniform grid $t_n = t_0 + nh, n = 0, 1, ..., N, Nh = T - t_0$, they take the form:

$$\begin{aligned} Y_i^{[n]} &= h \sum_{j=1}^{m} a_{ij} f\left(Y_j^{[n]}\right) + \sum_{j=1}^{r} u_{ij} y_j^{[n-1]}, \quad i = 1, 2, ..., m, \\ y_i^{[n]} &= h \sum_{j=1}^{m} b_{ij} f\left(Y_j^{[n]}\right) + \sum_{j=1}^{r} v_{ij} y_j^{[n-1]}, \quad i = 1, 2, ..., r, \end{aligned} \tag{2}$$

where m and r are the numbers of the internal and external stages, respectively, and the coefficients are obtained by evaluating suitable polynomials. By means of collocation, the solution is approximated by a piecewise polynomial in $[t_0, T]$.
The implicitness of (2) is strictly connected to the structure of the matrix $A = [a_{ij}]_{i,j=1,...,m}$ so we compare different methods considering A as:

- full matrix: the system is fully implicit;
- lower triangular matrix: the equations are solved in m successive stages, with only a k-dimensional system to be solved at each stage;
- singly triangular matrix: in solving the nonlinear systems by means of Newton-type iterations, it is possible to use repeatedly the stored LU factorization of the jacobian;
- diagonal matrix: the system can be solved in parallel.

For these methods order conditions are provided and numerical results are compared.

References

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