Preface

... What we need is imagination.
Richard Feynman

This book deals with the numerical solution of differential problems within the framework of *Geometric Integration*, a branch of numerical analysis which aims to devise numerical methods able to reproduce, in the discrete solution, relevant geometric properties of the continuous vector field. Among them, a paramount role is played by the so called *constants of motion*, which are physical quantities that are conserved along the solution trajectories of a large set of differential systems, named *Conservative Problems*. In particular, the major emphasis will be on Hamiltonian systems, though more general problems will be also considered.

For canonical Hamiltonian problems, the most important constant of motion is the Hamiltonian function itself, which is often referred to as *the energy* of the system. For this reason, methods which are able to conserve the Hamiltonian are usually named *energy-conserving*. This book is meant to be a thorough, though concise, introduction to energy-conserving Runge-Kutta methods. The key tool exploited to devise these methods is what we have called *discrete line integral*: roughly speaking, one imposes energy conservation by requiring that a discrete counterpart of a line integral vanish along the numerical solution regarded as a path in the phase space.

This basic tool may be easily adapted to handle the conservation of multiple invariants. With the term *Line Integral Methods*, we collect all the numerical methods whose definition relies on the use of the line integral.

The material is arranged in order to provide, to the best of our ability, a quite friendly walk across the subject, yet still giving enough details to allow a concrete use of the methods, with a number of examples of applications. To this end, related Matlab software is downloadable at the homepage of the book (see Section A.2). On the other hand, the material is meant to be self-contained: only a basic knowledge about numerical quadrature and Runge-Kutta methods is assumed.

The book is organized in six chapters and one appendix:

1. Chapter 1 contains a primer on line integral methods, and provides a basic introduction to Hamiltonian problems and symplectic methods.

2. Chapter 2 contains the description of a number of Hamiltonian problems, which are representative of a variety of applications. Most of the problems will be later used as workbench for testing the methods.

3. Chapter 3 contains the core of the theoretical results concerning the main instance of Line Integral Methods, i.e., the class of energy-conserving Runge-Kutta methods, also named Hamiltonian Boundary Value Methods (HBVMs), specifically devised for the Hamiltonian case.

4. Chapter 4 addresses, in great detail, the issue of the actual implementation of HBVMs, which is paramount, in order to recover in the numerical solution what expected from the theory. The problems presented in Chapter 2 are used to assess the methods.

5. Chapter 5 deals with the application of HBVMs to handle the numerical solution of Hamiltonian partial differential equations, when the space-variable belongs to a finite domain and appropriate boundary conditions are specified.

6. Chapter 6 sketches a number of generalizations of the basic energy-conserving methods: the extension to the case of multiple invariants; the case of general conservative problems; the numerical solution of Hamiltonian boundary value problems.

7. Appendix A, at last, contains some background material concerning Legendre polynomials, along with a brief description of the Matlab software implementing the methods, which has been made available on the internet.

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