Asymptotic Analysis of Fluid Equations

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Nonlinear PDEs describing the fluid motion makes a long list, most of which date back to the 19th century or earlier. Among them are the Boltzmann equation, the Navier-Stokes and Euler equations, compressible and incompressible, to mention a few. The Newton equation should be also included in the list as a microscopic fluid equation which consider the fluid to be a many-particle system. On the other hand, the Navier-Stokes and Euler equations are macroscopic fluid equations regarding the fluid as a continuum, and the Boltmann equation is in-between.

Apart from the Newton equation, they are all nonlinear PDEs, but in quite different classes from each other both in the type as PDE and in the nonlinear structure. This has been raising a variety of challenging and interesting mathematical problems with important and fruitful results. The fluid equations have been and will remain a rich source for the progress of the theory of nonlinear PDEs.

Obviously, any of these equations can be applied to one and the same fluid, which means that they must be closely interrelated to one another. In physics, the following asymptotic diagram has been known for a long time. Mathematically, this provides a num-

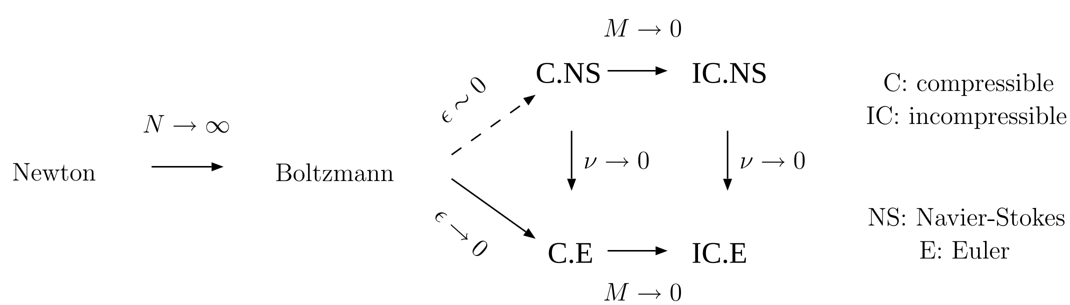

Figure 1: N: number of particles, ϵ: mean free path, ν: viscosity, M: Mach number

ber of highly sophisticated problems of singular parturbation. However, it is only in these decades that the mathematical proof of this diagram has become possible. Furthermore, with a recent progress of the multi-scale analysis, it also became possible to establish direct links (not via compressible macroscopic fluid equations) between the Boltzmann equations and the incompressible macroscopic (Euler and Navier-Stokes) equations, which are missing in the classical diagram Fig.1. The study of these asymptotic relations have revealed also the mathematical mechanism of the outbreak of the initial layer.

In the course of lectures, we will present these recent results, including the topics on the Cauchy Kovalevskaya technique for the Boltzmann-Grad limit of the Newton equation, the multi-scale analysis giving the compressible and incompressible limits of the Boltzmann equation and the analysis of their initial layers.