Equivariant derived category and representation of semisimple Lie groups

Masaki Kashiwara

By the work of Harish-Chandra, we can study the representations of the real reductive groups $G_{\mathbb{R}}$ by looking at their Harish-Chandra modules, which allows us to study representations in an algebraic way. Then, by results of Beilinson-Bernstein, we can study the Harish-Chandra modules by $\mathcal{D}$-modules on the flag varieties X, which leads us to a geometric study of K-orbits of the flag varieties. On the other hand, Matsuki correspondence gives a one-to-one correspondence between the K-orbits and the $G_{\mathbb{R}}$-orbits, and more generally there exists a one-to-one correspondence between the K-equivariant sheaves and $G_{\mathbb{R}}$-equivariant sheaves by Mirkovic-Vilonen-Uzawa.

In this series of lectures, I will present another method which connects representations of $G_{\mathbb{R}}$ with $G_{\mathbb{R}}$-equivariant sheaves on the flag manifold and explain the commutative diagram: