FLOWS, GENERALIZED DERIVATIVES, VARIATIONS AND NECESSARY CONDITIONS FOR AN OPTIMUM FOR CURVE MINIMIZATION PROBLEMS

H. SUSSMANN, Rutgers University.

ABSTRACT: Since the Pontryagin Maximum Principle was discovered in the 1950's, two competing approaches have been developed for proving extensions of the original result to theorems yielding stronger conclusions (for example, high-order conditions), or valid under weaker hypotheses (for example, right-hand sides that are Lipschitz rather than of class C^1) or both. The first, "primal," method, consists of working systematically with families of vector fields, making "needle variations," differentiating the corresponding endpoint map, and applying open mapping conditions based on some version of the Brouwer fixed point theorem. The second, "dual" approach, takes as its starting point the classical calculus of variations, and reduces the optimal control problems for vector fields and differential inclusions to problems of the more classical kind, by means of penalization methods in which violations of the constraints are prevented by addinga large penalty term in the cost functional. In the 1970's and 1980's, a lot of the progress in the non-smooth direction was made by means ofdual methods, while the primal approach yielded better results (with extra conclusions) for smooth systems but appeared not to lend itself to the study of non-smooth problems. This situation changed in the early 1990's, with the pioneering work of S. Lojasiewicz Jr., who showed how to get strong nonsmooth results with a primal technique. In the course, we will present a general primal methodology that has evolved out of Lojasiewicz's idea, and has led to a unified approach that applies to much larger classes of smooth and very non-smooth problems, including those that involve differential inclusions. This methodology is based on

(a) using vector fiels that are not necessarily continuous (following ideas of Alberto Bressan) in order to overcome the fact that the set-valued maps that occur in differential inclusion systems typically do not admit continuous selections, and to make sure that there are sufficiently many nice selections to make it possible to reduce all differential inclusion systems to systems of vector fields,

(b) relying systematically on regularity properties of flows rather than on properties of the vector fields that generate them, so as to exploit the fact that usually flows are more regular than their infinitesimal generators,

(c) considering abstract variations (of the kind studied by Krener, Knobloch, Bianchini, Stefani, Kawski, and others) rather than just the classical needle variations,

and, most importantly,

(d) using notions of derivative other than the classical one to differentiate the endpoint maps.

In particular, the course will give an axiomatic description of the notion of a "generalized differentiation theory" as a multifunctor between some appropriate categories, and provide a detailed presentation of one such theory—the "generalized differential quotients"—that is both fairly simple and of very broad applicability.

REFERENCES:

A Papers by H. J. Sussmann

All these papers can be downloaded from H. Sussmann's Web page,

http://www.math.rutgers.edu/ sussmann/currentpapers.html

In the list below, the number given for each item is the number under which it is listed in the Web page.

72. HIGH-ORDER OPEN MAPPING THEOREMS. In "Directions in Mathematical Systems Theory and Optimization (a selection of papers dedicated to Anders Lindquist)," Anders Rantzer and Chris Byrnes Eds., Springer Verlag, 2002. (The paper is 27 pages long.

70. PATH-INTEGRAL GENERALIZED DIFFERENTIALS. In "Proceedings of the 2002 Conference on Decision and Control," held in Las Vegas, Nevada, December 2002. (The paper that actually appeared in the conference proceedings was only 6 pages long, and omitteds important details, such as the very long proof of the "error correction property" for polyhedral cones. The 13 pages long Web page version is much more detailed and contains the full details of all the proofs.)

69. WARGA DERIVATE CONTAINERS AND OTHER GENERALIZED DIFFERENTIALS. In "Proceedings of the 2002 Conference on Decision and Control," held in Las Vegas, Nevada, December 2002. (The paper is 5 pages long.)

67. A LOCAL SECOND- AND THIRD-ORDER MAXIMUM PRINCIPLE. In "Proceedings of the 2002 American Control Conference," held in Anchorage, Alaska, May 8-10, 2002. (The paper is 6 pages long.)

66. NEEDLE VARIATIONS AND ALMOST LOWER SEMICONTINUOUS DIFFERENTIAL INCLUSIONS. Published in "Set valued analysis," 2002. (The paper is 59 pages long.)

62. NEW THEORIES OF SET-VALUED DIFFERENTIALS AND NEW VERSIONS OF THE MAXIMUM PRINCIPLE OF OPTIMAL CONTROL THEORY. In "Nonlinear Control in the Year 2000," A. Isidori, F. Lamnabhi-Lagarrigue and W. Respondek Eds.; Springer-Verlag, 2000; pages 487-526.

59. RESULTATS RECENTS SUR LES COURBES OPTIMALES. In "Quelques aspect de la theorie du controle, Journee Annuelle de la Societe Mathemathique de France (SMF)," Publications de la SMF, Paris, 2000, pp. 1-52.

54. A MAXIMUM PRINCIPLE FOR HYBRID OPTIMAL CONTROL PROBLEMS. In "Proceedings of the 38th IEEE Conference on Decision and Control (Phoenix, AZ, Dec. 7-10, 1999)," IEEE Publications, New York, 1999, pp. 425-430.

53. A NONSMOOTH HYBRID MAXIMUM PRINCIPLE. In "Stability and Stabilization of Nonlinear Systems," D. Aeyels, F. Lamnabhi-Lagarrigue and A. J. van der Schaft Eds., Lecture Notes in Control and Information Sciences, no. 246, Springer-Verlag, pp. 325–354.

50. GEOMETRY AND OPTIMAL CONTROL. In "Mathematical Control Theory (a book of essays in honor of Roger W. Brockett on the occasion of his 60th birthday)," J. Baillieul and J. C. Willems, Eds., Springer-Verlag, New York, 1998, pages 140-198. The text is 59 pages long. The date of the current Web version is October 3, 1998. In this version, some typographical errors discovered after the book went to press have been corrected.

B. Other background material (books):

Berkovitz, L. D., "Optimal Control Theory." Springer-Verlag, New York, 1974.

Clarke, F. H., "Optimization and Nonsmooth Analysis." Second Edition, Classics in Applied Mathematics Vol. 5, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1990.

Clarke, F. H., Y. Ledyaev, R. Stern, and P. Wolenski, "Nonsmooth Analysis and Control Theory." Springer, New York, 1998.

Pontryagin, L. S., V.G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mischenko, "The Mathematical Theory of Optimal Processes." Wiley, New York, 1962.