Spatial Point Processes and their Applications

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A spatial point process is a model for a random pattern of points in d-dimensional space (usually $d = 2$ or $d = 3$ in applications). These processes play a special role in stochastic geometry as the building blocks of more complicated random set models (such as the Boolean model) [4, 3] and they also serve as instructive simple examples of random sets. In the related field of Spatial Statistics [1], point processes are used directly as statistical models of patterns of points or point-like objects.

These lectures will introduce some basic techniques for constructing, manipulating and analysing spatial point patterns. The lecture titles are:

1. Point processes
2. Moments and summary statistics
3. Conditioning
4. Modelling and statistical inference

In Lecture 1. Point processes we motivate and define point processes, construct examples (especially the *Poisson process* [2]), and analyse important properties of the Poisson process. There are different ways to construct and characterise a point process (using finite-dimensional distributions, vacancy probabilities, capacity functional, or generating function). An easier way to construct a point process is by transforming an existing point process (by thinning, superposition, or clustering) [4]. In the computer exercises we will generate simulated realisations of many spatial point processes using these techniques, and analyse them using vacancy probabilities (or 'empty space functions').

In Lecture 2. Moments and summary statistics we describe the analogue, for point processes, of the expected value and higher moments of a random variable. These quantities are useful in theoretical analysis and in statistical inference. The 'intensity' or first moment of a point process is the analogue of the expected value of a random variable. *Campbell's formula* is an important result for the intensity. The 'second moment measure' is related to the variance or covariance of random variables. The *"K function"* and *pair correlation* are second-moment properties which have many applications in the statistical analysis of spatial point patterns [1, 4]. The second-moment properties of some point processes will be derived. In the computer exercises we will compute statistical estimates of the K function from spatial point pattern data sets.

In Lecture 3. Conditioning we explain how to condition on the event that the point process has a point at a specified location (even though this event has probability zero). This leads to the *Palm distribution* of the point process, and the related *Campbell-Mecke formula* [3]. These tools allow us to define new characteristics of a point process, such as the nearest neighbour distance function G. A dual concept is the *conditional intensity* which provides many new results about point processes. In the computer exercises we compute statistical estimates of the G and J functions from spatial point pattern data sets.

In Lecture 4. Modelling and statistical inference we consider point processes in a bounded region of space. Under this restriction it is possible to define point processes by writing down their probability densities. Statistical models for finite point processes are surveyed. We describe techniques for fitting such models to data [1]. In the computer exercises, we apply the method of maximum pseudolikelihood to fit point process models to real spatial point pattern data sets.

References

[1] P.J. Diggle. *Statistical Analysis of Spatial Point Patterns.* Arnold, second edition, 2003.

[2] J.F.C. Kingman. *Poisson Processes.* Oxford University Press, 1993.

[3] D. Stoyan, W.S. Kendall, and J. Mecke. *Stochastic Geometry and its Applications.* John Wiley and Sons, Chichester, second edition, 1995.

[4] D. Stoyan and H. Stoyan. *Fractals, random shapes and point fields.* Wiley, 1995.