Integral Geometric Tools for Stochastic Geometry

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We give an introduction to the parts of integral geometry which are needed in stochastic geometry, in particular in the study of Boolean models. Starting from this introduction, we present various recent extensions, refinements, and generalizations. The titles of the lectures (of 90 minutes each) are:

1. From hitting probabilities to kinematic formulae
2. Localizations and extensions
3. Translative integral geometry
4. Intersection formulae without invariance

The general theme are measures on spaces of geometric objects, combined with intersections of such objects. For example, the principal kinematic formula provides an explicit expression for the invariant measure of the set of all rigid motions bringing a convex body into a position where it has nonempty intersection with a fixed convex body. This classical result serves as an introduction: it leads in a natural way to the Steiner formula, to intrinsic volumes and their properties, to Hadwiger's characterization theorem, and this in turn yields a quick proof for the complete system of kinematic formulae for the intrinsic volumes, as needed in the theory of isotropic and stationary Boolean models. As soon as invariant measures on spaces of flats are available, one can deduce corresponding intersection formulae (Crofton formulae for convex bodies) and Hadwiger's general integral-geometric theorem.

The second lecture aims at extensions of the Steiner formula and the kinematic formula in various directions. A local version of the Steiner formula leads to curvature measures, and these again satisfy kinematic formulae. Various possibilities of extending Steiner formula and kinematic formulae to non-convex sets will be discussed.

To serve the purposes of the stochastic geometry of homogeneous but non-isotropic structures, an integral geometry for the translation group has to be developed. The third lecture presents the recent developments in this direction. Naturally, the translative counterparts to the kinematic intersection formulae and their iterations for curvature measures become more complicated, and they involve an additional set of mixed measures. Further recent results of translative integral geometry concern support functions or general simple valuations.

Going one step further in weakening the invariance assumptions, we study, in the fourth lecture, measures on spaces of flats which need not have any invariance property, but are required to satisfy intersection formulae of Crofton type for lower dimensional areas. In particular, the association of certain (generalized) zonoids with measures on the space of hyperplanes is exploited here in various ways. For example, we show how non-stationary Poisson hyperplane processes, satisfying a certain regularity property, induce via intersection probabilities a projective Finsler metric and how, conversely, in projective Finsler spaces integral-geometric Crofton formulae can be established.

Selected Literature:

R. Schneider and W. Weil, *Integralgeometrie.* Teubner, Stuttgart 1992.

D. Hug and R. Schneider, Kinematic and Crofton formulae of integral geometry: recent variants and extensions. In *Homenatge al professor Lluís Santaló i Sors* (C. Barceló i Vidal, ed.), Universitat de Girona, 2002, pp. 51 – 80.

D. Hug and R. Schätzle, Intersections and translative integral formulas for boundaries of convex bodies. *Math. Nachr.* **226** (2001), 99–128.

W. Weil, Iterations of translative integral formulae and non-isotropic Poisson processes of particles. *Math. Z.* **205** (1990), 531–549.

R. Schneider, Crofton formulas in hypermetric projective Finsler spaces. *Arch. Math.* **77** (2001), 85 – 97.