CIME COURSE

Ricci Flow and Geometric Applications

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Course Directors

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Lectures

Gérard Besson (Grenoble)
Ricci flow in higher dimension.

Abstract: The course will focus on the impact of the Ricci flow technique on the structure of positively curved manifolds. The central result is the sphere theorem proved by Brendle and Schoen. It shows that a closed simply-connected manifold which is pointwise strictly 1/4–pinched is a sphere. It relies on ideas similar to the work of Böhm and Wilking showing that a closed manifold with positive curvature operator is a sphere. The notion of positive isotropic curvature will be discussed. The main issue is to discuss what kind of curvature conditions are preserved by the Ricci flow.
Extensions appearing after this summary is written may be included.

• C. Böhm and B. Wilking, Manifolds with positive curvature operators are space forms. Ann. of Math. (2) 167, no. 3, 1079–1097, 2008.
• S. Brendle and R. M. Schoen, Classification of manifolds with weakly $1/4$–pinched curvatures. Acta Math. 200, no. 1, 1–13, 2008.

Michel Boileau (Toulouse)
Collapsing phenomena in dimension 3 and their application to geometrization.

Abstract: Assuming the results of more analytic nature, we will discuss some global topological aspects entering in the geometrization of 3–manifolds by means of Ricci Flow.

• L. Bessières, G. Besson, M. Boileau, S. Maillot and J. Porti, Weak collapsing and geometrization of aspherical 3–manifolds. ArXiv Preprint Server, 2007.
• J.–P. Otal, Thurston's hyperbolization of Haken manifolds'. Surveys in differential geometry, Vol. III Cambridge, MA, 77–194, Int. Press, Boston, MA, 1998.
• Michel Boileau, Bernhard Leeb and J. Porti, Geometrization of 3–dimensional orbifolds, Ann. of Math. (2) 162, No. 1, 195–290, 2005.

Carlo Sinestrari (Roma Tor "Vergata")
The structure of singularities of the Ricci flow in dimension three.

Abstract: In these lectures we first review the basic properties of the Ricci flow and the fundamental estimates of the theory, such as Hamilton's Harnack differential inequality, Hamilton–Ivey's pinching estimate and Perelman's no collapsing result. Then we present Perelman's analysis of kappa–solutions and the canonical neighborhood property which gives a full description of the singular behavior of the solutions in dimension 3.

• R.S. Hamilton, The formation of singularities in the Ricci flow. Surveys in differential geometry, Vol. II, 7–136, Internat. Press, Cambridge, MA, 1995.
• G. Perelman, The entropy formula for the Ricci flow and its geometric applications. ArXiv Preprint Server, 2002.
• G. Perelman, Ricci flow with surgery on three–manifolds. ArXiv Preprint Server, 2003.

Gang Tian (Princeton)
Kähler–Ricci flow and geometric applications.

Abstract: This course concerns Kähler–Ricci flow and its connection to the classification of projective manifolds. I will discuss basic tools in and recent progress on Kähler–Ricci flow. Some open problems and applications will be also presented.

• R. Hamilton, The formation of singularities in the Ricci flow. Surveys in Differential Geometry, vol. II, 1993.
• J. Song and G. Tian, The Kähler–Ricci flow on surfaces of positive Kodaira dimension. Invent. Math. 170, 2007.
• N. Sesum and G. Tian, Bounding scalar curvature and diameter along the Kähler Ricci flow (after Perelman). J. Inst. Math. Jussieu 7, 2008.
• G. Tian and X.H. Zhu, Convergence of Kähler-Ricci flow. J. Amer. Math. Soc. 20, 2007.