Matteo Focardi's Curriculum Vitae

May 10, 2012

Personal Data

born: December 27th, 1973, in Florence. Italian citizen.

current position: "Ricercatore" (assistant Professor) in Analysis, University of Florence.

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Education

1992-1996: *Laurea in Matematica* at Università di Firenze, final grade: summa cum laude. Advisor Prof. Elvira Mascolo, thesis title "*Lower Semicontinuity of vector valued functionals in Orlicz-Sobolev spaces*", defended on 25.10.96;

1997: *INdAM junior scholarship*, courses held at Università di Roma "La Sapienza";

1998-2000: *Perfezionamento in Matematica* at Scuola Normale Superiore di Pisa, final grade: summa cum laude. Advisor Prof. Luigi Ambrosio, thesis title "*Variational Approximation of Vectorial Free-Discontinuity Problems: the Discrete and Continuous case*", defended on 14.06.02.

Positions

2000-present: *"Ricercatore"* (assistant Professor) at the Department of Mathematics of the University of Florence, since 1.09.00;

2003: postdoc position at *Max Planck Institut für Mathematik in den Naturwissenschaften* in Leipzig (D), from 16.03.03 to 27.07.03.

Teaching

Courses

2000/01, 02/03, 04/05, 06/07, 08/09, 10/11: teaching assistant of *Analisi Matematica 1*, first year analysis for undergraduate students in Physics;

2001/02, 02/03, 04/05, 06/07, 07/08: teaching assistant of *Analisi Matematica 2A*, second year analysis (one semester) for undergraduate students in Physics;

2001/02, 03/04, 05/06, 07/08, 09/10, 11/12: teaching assistant of *Analisi Matematica 2B*, second year analysis (one semester) for undergraduate students in Physics;

2003/04: teaching assistant of *Matematica*, basic calculus for undergraduate students in Biotechnologies;

2004/05: teacher of *Calculus of Variations*, advanced fourth year course for undergraduate students in Mathematics;

2005/06, 06/07, 07/08, 09/10: teacher of *Complementi di Analisi*, advanced third year course for undergraduate students in Physics;

2007/08: teacher of *An introduction to* Γ*-convergence*, PhD course in Mathematics.

2009/10: teaching assistant of *Matematica*, basic calculus for undergraduate students in Biology;

2010/11, 11/12: teacher of *Matematica*, basic calculus for undergraduate students in Biotechnologies.

Undergraduate Students

Claudia Bergesio, BA thesis: "Optimal Young's inequality for convolutions", defended on June 2006;

Claudia Bergesio, Master Degree Thesis: "*Un problema di omogenizzazione con vincoli sulle tracee*", defended on April 2009;

Claudia Berloco, BA thesis: "Il teorema di decomposizione polare per mappe vettoriali" defended on October 2009.

Research

Papers[1]

1. M. Focardi, *Semicontinuity of vectorial functionals in Orlicz-Sobolev spaces*, Rend. Ist. Mat. Univ. Trieste, vol. **XXIX** (1997), 141–161;

2. R. Alicandro - M. Focardi - M.S. Gelli, *Finite-difference approximation of energies in fracture mechanics*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **29** (2000), 671–709;

3. M. Focardi - E. Mascolo, *Lower semicontinuity of quasiconvex functionals with non-standard growth*, Journal of Convex Analysis **8** (2001), 1–22;

4. M. Focardi, *On the variational approximation of free-discontinuity problems in the vectorial case*, Math. Models Methods Appl. Sci. **11** (2001), 663–684;

5. R. Alicandro - M. Focardi, *Variational approximation of free-discontinuity energies with linear growth*, Comm. Cont. Math., Vol. 4, No. **4** (2002), 685-723;

6. B. Bianconi - M. Focardi - E. Mascolo, *Existence results for quasiconvex functionals with non-standard growth conditions*, Ann. Mat. Pura Appl. (4) **180** (2002), 493–510;

7. M. Focardi, *Variational Approximation of Free-Discontinuity Problems: the Discrete and Continuous Case*, Tesi di Perfezionamento, Relatore Prof. L. Ambrosio, Scuola Normale Superiore di Pisa, 2002;

8. M. Focardi - M.S. Gelli, *Approximation results by difference schemes of fracture energies: the vectorial case*, NoDEA Nonlinear Differential Equations Appl., Vol. 10, No. **4** (2003), 469–495;

9. M. Focardi - M.S. Gelli, *Asymptotic analysis of the Mumford-Shah functional in periodically perforated domains*, Interfaces and Free Boundaries **9** (2007), 107–132;

10. M. Focardi - A. Garroni, *A $1D$ macroscopic phase field model for dislocations and a second order Γ-limit*, SIAM Multiscale Model. Simul. **6** (2007), no. 4, 1098–1124.

11. M. Focardi - M.S. Gelli, *Relaxation of free-discontinuity energies with obstacles*, Esaim COCV, **14** (2008), No. 4, 879-896;

12. M. Focardi - P.M. Mariano, *Convergence of asynchronous variational integrators in linear elastodynamics*, Internat. J. Numer. Methods in Engrg., **75** (2008), 755–769;

13. M. Focardi - P.M. Mariano, *Discrete dynamics of complex bodies with substructural dissipation: variational integrators and convergence*, Discr. Contin. Dyn. Syst., **11** 1 (2009), 109–130;

[1] Most of the papers are downloadable at `http://cvgmt.sns.it/people/focardi/`

14. M. Focardi - M.S. Gelli - M. Ponsiglione, *Fracture mechanics in perforated domains: a variational model for brittle porous media*, Math. Models Methods Appl. Sci., **19** 11 (2009), 2065-2100;

15. M. Focardi, *Homogenization of random fractional obstacle problems via* Γ*-convergence*, Comm. Partial Differential Equations **34** (2009), 1607–1631;

16. M. Focardi, M.S. Gelli, G. Pisante, *On a* 1*-capacitary type problem in the plane*, Commun. Pure Appl. Anal. **9** (2010), no. 5, 1319–1333;

17. M. Focardi, *Aperiodic fractional obstacle problems*, Adv. Math. 225 (2010), 3502–3544;

18. C. De Lellis, M. Focardi, E.N. Spadaro, *Lower semicontinuous functionals for Almgren's multiple valued functions*, Annales Academiæ Scientiarum Fennicæ Mathematica **36** (2011), 1–18;

19. M. Barchiesi, M. Focardi, *Homogenization of the Neumann problem in perforated domains: an alternative approach*, Calc. Var. & PDEs published on line DOI 10.1007/s00526-010-0387-2;

20. M. Focardi, *Vector-valued obstacle problems for non-local energies*, Discr. Contin. Dyn. Syst. B **17** (2012), 487–507;

21. M. Focardi, E.N. Spadaro, *An intrinsic approach to manifold constrained variational problems*, accepted for publication on Annali Matematica Pura e Applicata.

22. C. De Lellis, M. Focardi, *Density lower bound estimates for local minimizers of the Mumford-Shah energy in* 2*d*, submitted paper.

23. C. De Lellis, M. Focardi, *Higher integrability of the gradient for minimizers of the* 2*d Mumford-Shah energy*, submitted paper.

24. M. Focardi, Γ*-convergence: a tool to investigate physical phenomena across scales*, accepted for publication on MMAS.

Conference Proceedings

1. *Convergence of asynchronous variational integrators in linear elastodynamics*, Proceedings AIMETA 2007.

2. *Asymptotic analysis and relaxation of Mumford-Shah type energies with obstacles*, PAMM, Proc. Appl. Math. Mech. 7, 1150807-1150808 (2007)/DOI 10.1002/pamm.200700448.

3. *Convergence of AVI's for the linear elastodynamics of simple bodies*, Proceedings IACM/ECCOMAS 2008.

Talks

"Approssimazione alle differenze finite di energie della meccanica delle fratture", Dipartimento di Matematica "L. Tonelli" Università di Pisa, 17.02.2000

"Approssimazione alle differenze finite di energie della meccanica delle fratture", workshop *Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza*, Levico Terme 31.01-3.02.2000

"Problemi a discontinuità libera per funzionali a crescita lineare: un'approssimazione variazionale", Dipartimento di Matematica "U. Dini" Università di Firenze, Firenze 14.02.2001

"Variational Approximation of free-discontinuity problems via functionals involving the L^1-norm of the gradient", workshop *Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza*, Levico Terme 29.03.2001

"Approximation results by difference schemes of fracture energies: the vectorial case", workshop *Secondo incontro Italia-Argentina di Matematica Pura e Applicata*, Buenos Aires (AR) 16.12.2002

"Variational Approximation of free-discontinuity problems via functionals involving the L^p-norm of the gradient", Arbeitsgemeinschaft Mikrostrukturen del "Max Planck Institut für Mathematik in den Naturwissenschaften", Leipzig (D) 15.04.2003

"Some Remarks on Unilateral Pure Obstacle Problems for Free-Discontinuity Functionals", Séminaire d'Analyse *Universitè Marne-la-Vallèe*, Parigi (FR) 7.12.2004

"Problemi a discontinuità libera con ostacolo: il caso Mumford-Shah su domini perforati", workshop *Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza*, Levico Terme 31.01.2005

"Asymptotic analysis of the Mumford-Shah functional in periodically perforated domains", Arbeitsgemeinschaft Analysis Institut für Mathematik Universität Zürich 12.01.2006

"Asymptotic analysis of Mumford-Shah type energies in periodically perforated domains", SIMAI workshop 2006 (session M23 "Models of multi-scale phenomena in simple and complex bodies")

"Asymptotic expansion for a non-local phase-field theory for dislocations", SIMAI workshop 2006 (session M37 "Variational Methods and Applications")

"Sviluppo asintotico di un modello variazionale per le dislocazioni nel caso unidimensionale", workshop *Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza*, Levico Terme 8.02.2007

"Espansione asintotica di un modello variazionale per le dislocazioni nel caso unidimensionale", Seminari del Dipartimento di Matematica "U. Dini" Università di Firenze, Firenze 23.02.2007

"Asymptotic analysis and relaxation of Mumford-Shah type energies with obstacles, ICIAM 2007, Minisymposium IC/MP/092/N/697 "Geometric and stochastic aspects of the mechanics of complex bodies";

"Convergence of asynchronous variational integrators in linear elastodynamics", XVIII Congresso AIMETA, Brescia 12.09.2007

"Homogenization of free-discontinuity energies in periodically perforated domains", PDE Seminar Univ. Minnesota, Minneapolis 7.05.2008

"Homogenization of Mumford-Shah type energies in periodically perforated domains", SIAM Conference 2008 "Mathematical aspects on materials science", Philadelphia 12.05.2008

"Free-discontinuity energies in periodically perforated domains", Center for Nonlinear Analysis Seminars, Carnegie Mellon University, Pittsburgh 28.05.2008

"The effect of micro-cracks on bulk & surface energies", Workshop on Elliptic and Parabolic Equations and Systems, Naples, 17.06.2008

"Convergence of AVIs for the Linear Elastodynamics of Simple Bodies", IACM/WCCM8 (8th World Congress in Computational Mechanics), Venice, 3.07.2008,

"Omogenizzazione per problemi di ostacolo frazionari", workshop *Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza*, Levico Terme 9.02-13.02.2009

"Lower semicontinuous functionals for Almgren's Q-valued maps", Seminari del Dipartimento di Matematica "U. Dini" Università di Firenze, Firenze 18.12.2009

"Homogenization of fractional obstacle problems", INdAM workshop on vector-valued mappings and systems of PDE's, Rome 21.05.2010

"Omogenizzazione di ostacoli frazionari", Seminari del Dipartimento di Matematica Applicata "G. Sansone" Università di Firenze, Firenze 31.05.2010

"Aperiodic fractional obstacle problems", Fifth Meeting on Applied Mathematics and Calculus of Variations, Rome 12.06.2010

"Fractional obstacle problems", Meeting SIMAI 2010, Cagliari 21.06.2010

"Homogenization of fractional obstacle problems", Universitá di Roma Tor Vergata, Rome 15.03.2011

"An intrinsic approach to manifold constrained variational problems", EPFL Lausanne, 8.04.2011

"An intrinsic approach to manifold constrained variational problems", Center for Nonlinear Analysis Seminars, Carnegie Mellon University, Pittsburgh 10.05.2011

"Fractional obstacle problems", Centro De Giorgi school on 'Mathematical Principles for Advances in Continuum Mechanics', Pisa 7.11.2011

"Maggiore integrabilità del gradiente per minimi locali del funzionale di Mumford-Shah in dimensione 2", workshop *Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza*, Levico Terme 5.02-10.02.2012.

"Questioni di regolarità per i minimi locali del funzionale di Mumford-Shah in dimensione 2", Università di Bologna "Seminario Pini", Bologna 15.03.2012

"Regularity issues for local minimizers of the Mumford-Shah energy in 2d", SISSA, Trieste 28.03.2012

"Regularity issues for local minimizers of the Mumford-Shah functional in 2d", Max Planck Institut für Mathematik in den Naturwissenschaften in Leipzig (D), 10.04.2012

"Regularity issues for local minimizers of the Mumford-Shah energy in 2d", Dipartimento di Matematica "L. Tonelli", Università di Pisa, 9.05.2012

Visits

Universität Zürich, Zürich (CH), invited by Prof. Camillo De Lellis, 23.05-11.06.04;

Universitè Marne-la-Vallèe, Paris (FR), invited by Prof. Marco Cannone, 6-10.12.04;

Universität Zürich, Zürich (CH), invited by Prof. Camillo De Lellis, 3-10.07.05;

Universität Zürich, Zürich (CH), invited by Prof. Camillo De Lellis, 8.01-14.01.06;

University of Minnesota (School of Mathematics), Minneapolis (USA), invited by Prof. Vladimir Sverak, 3.05-10.05.08;

Carnegie Mellon University (Center for Nonlinear Analysis), Pittsburgh (USA), invited by Prof. Irene Fonseca, 15.05-7.06.08;

Universität Zürich, Zürich (CH), invited by Prof. Camillo De Lellis, 7-12.12.08;

Universität Zürich, Zürich (CH), invited by Prof. Camillo De Lellis, 26.04-16.05.09;

Universität Zürich, Zürich (CH), invited by Prof. Camillo De Lellis, 8.03-2.04.10;

Universitá di Roma Tor Vergata, Rome, invited by Prof. Enrico Valdinoci 14-16.03.2011;

EPFL, Lausanne (CH), invited by Prof. Bernard Dacorogna, 7-8.04.2011;

Universität Zürich, Zürich (CH), invited by Prof. Camillo De Lellis, 9-16.04.11;

Carnegie Mellon University (Center for Nonlinear Analysis), Pittsburgh (USA), invited by Prof. Irene Fonseca and Prof. Giovanni Leoni, 30.04-14.05.11;

SISSA, invited by Prof. G. Dal Maso, 27-30.03.12;

Max Planck Institut für Mathematik in den Naturwissenschaften in Leipzig (D), invited by Dr. Emanuele Spadaro, 9-14.04.12.

Attended Workshops and Schools

Scuola Matematica Interuniversitaria, Perugia 31.07.97-31.08.97;

Nonlinear analysis function spaces and applications, Praga (CZ) 31.05.98-6.06.98;

III International School on Calculus of Variations, Pisa 28.09.98-2.10.98;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 28.02.99-5.03.99;

Nonlinear Differential Equations: Oszillations, Singularities and Microstructures, Oberwolfach (D) 23-29.05.99;

Partial Differential Equations and Calculus of Variations, Isola d'Elba 24-30.10.99;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 31.01.00-3.02.00;

School on Mathematical Problems in Image Processing, Trieste 4-22.09.00;

Selected issues in the Mechanics of Cristalline Solids, Padova 2-5.10.00;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 25-29.03.01;

CNA Summer School, Pittsburgh (USA) 25.05.01-9.06.01;

Homogeneization techniques and asymptotic methods for problems with multiple scales: Part I, Torino 17-21.09.01;

La Matematica di Ennio De Giorgi, Pisa 24-27.10.02;

Homogeneization techniques and asymptotic methods for problems with multiple scales: Part II, Roma 3-5.12.01;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme febbraio 2002;

IV International School on Calculus of Variations, Pisa 20-25.05.02;

Oberwolfach Seminar: Mass Transportation Problems and Applications, Oberwolfach (D) 14-20.10.02;

Recent advances in Calculus of Variations and PDE's - A young researchers meeting, Pisa 7-9.11.02;

Secondo incontro Italia-Argentina di Matematica Pura e Applicata, Buenos Aires (AR) 16-21.12.02;

Arbeitsgemeinschaft mit aktuellem Thema: Convex Integration, Oberwolfach (D) 30.03.03-5.04.03;

School on Concentration Phenomena for Variational Problems, Roma 1-5.09.03;

Optimal Transport Theory and Applications, Pisa 9-12.10.03;

Analisi e Convessità, Cremona 23-24.01.04;

School on Geometric Evolution Problems, Roma 26-28.01.04;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 2-6.02.04;

Variational Problems in Materials Science, Trieste 6-10.09.04;

Analysis and Applied Mathematics, Roma 20-24.09.04;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme (TN) 30.01-4.02.05;

Calculus of Variations and non linear partial differential equations, Corso CIME, Cetraro (CS) 27.06-2.07.05;

Analysis and Applied Mathematics II, Roma 12-16.09.05;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme (TN) 5-10.02.06;

Matematica nei Materiali, Workshop INdAM, Roma 3-7.04.06;

VIII Congresso SIMAI, Baia Samuele (RG) 22-26.05.06;

Analysis and Applied Mathematics III, Roma 5-9.06.06;

Variational and Differential Problems with Constraints, Venezia 18-20.09.06;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme (TN) 4-9.02.07;

Analysis and Applied Mathematics IV, Roma 11-15.06.07;

New Trends in Partial Differential Equations and Calculus of Variations, Cortona (AR) 6-12.05.07;

ICIAM 2007, 6th International Congress on Industrial and Applied Mathematics, Zürich (CH) 16-20.07.07;

XVIII Congresso AIMETA, Brescia 11-14.09.07;

Structures of the mechanics of complex bodies, Centro “E. De Giorgi”, Pisa 1-6.10.07;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 3-8.02.08;

Mathematical aspects on materials science - SIAM Conference 2008, Philadelphia (USA) 11-15.05.08;

CNA 2008 Summer School - Contemporary Topics in Nonlinear PDEs, Pittsburgh (USA) 29.05-7.06.08;

IACM/WCCM8 - 8th World Congress in Computational Mechanics, Venezia, 30.06-4.07.08;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 9.02-13.02.09;

Analysis and Applied Mathematics V, Roma 1-5.06.09;

Aspects of convexity, Firenze 27-28.11.2009;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 21.02-26.02.10;

INdAM workshop on vector-valued mappings and systems of PDE's, Rome 17-21.05.2010;

Meeting on Applied Mathematics and Calculus of Variations, Rome 10-12.06.2010;

X Congresso SIMAI, Cagliari 21-25.06.10;

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 7.02-11.02.11.

Calculus of Variations, Continuum Mechanics and Geometric Inequalities, Ischia 5-10.06.2011.

Mathematical Principles for Advances in Continuum Mechanics, Centro De Giorgi, Pisa 7-11.11.2011.

Calcolo delle Variazioni: Teoria Geometrica della Misura, Rilassamento, Gamma Convergenza, Levico Terme 5.02-10.02.12.

Miscellanous Scientific Activities

Joint Organizer, “Weekend di Calcolo delle Variazioni - Workshop a Firenze”, Florence 4-5.11.2005;

Joint Organizer, “Workshop on Calculus of Variations and PDEs”, Florence 23-24.01.2009;

Joint Organizer, Analysis Seminars of the Florence Math Department “Ulisse Dini” a.a. 08/09, 09/10, 10/11, 11/12;

Chairman of the symposium “Analytical and geometrical problems in Continuum Mechanics” within the X SIMAI Congress of the Italian Society for Industrial and Applied Mathematics, Cagliari (Italy), June 21-25 2010.

Guest editor with Prof. P.M. Mariano of the special issue “Analytical and Geometrical Problems in Continuum Mechanics”, DCDS-B, AIMS, scheduled for March 2012.

Jury member (examinateur) of the “Doctoral Thesis” “On a charaterization of the relaxation of a generalized Willmore functional” by G. Nardi , Université Pierre et Marie Curie, Laboratoire “Jacques-Louis Lions”, Paris, 12.12.2011.

Service as reviewer for the American Mathematical Society.

Service as referee for the following journals: *SIAM Journal on Mathematical Analysis, ESAIM COCV, DCDS-B, ZAMM, MMA, Physica D, IFB, Journal of Differential Equations, Applicable Analysis, Calc. Var.* & *PDEs, Comm. Partial Differential Equations, JMMA, ARMA, MMAS* .

Research Projects *(in which I had managing responsibilities)*

GNAMPA 2010 project ”Semicontinuità di funzionali definiti su mappe multivoche, vettoriali e problemi collegati“.

Research Interests

My research interests lie in the areas of Calculus of Variations, Geometric Measure Theory and Continuum Mechanics. In the following I will briefly resume the contents of my results.

References [1],[3],[6]: My Degree Thesis was focused on lower semicontinuity properties for integral functionals whose densities satisfy non-standard growth conditions, i.e. they have different polynomial growth from above and from below.

In particular, classical results for Sobolev spaces in the case of standard growth are extended to a wide class of energies whose densities are controlled in terms of a positive function Φ which is convex and superlinear at infinity: A N-function. The theory of Orlicz, Orlicz-Sobolev spaces is then needed.

The Orlicz spaces approach enables, for instance, to prove existence results for Dirichlet boundary value problems in the natural domain of definition of the functional endowed with a natural topology. Instead, the results in Sobolev spaces known in the literature would require lower semicontinuity properties in a stronger topology than that under which the functional is coercive being the function Φ in general bounded from below/above by two different powers.

References [2],[4],[5],[8]: During my PhD studies I worked on the variational approximation of free-discontinuity energies, whose prototype example is the Mumford-Shah functional of image reconstruction, i.e.

$$MS(u) = \int_\Omega |\nabla u|^p dx + \mathcal{H}^{n-1}(S_u).$$

Here $\Omega \subset \mathbf{R}^n$ is a given regular bounded open set, $p \in (1, +\infty)$, ∇u and S_u are, respectively, the (approximate) gradient and the set of (approximate) discontinuities of u. This kind of energies have been employed successfully also to give weak formulations of Fracture Mechanics theories for brittle hyperelastic media.

In the papers [2],[4],[5],[8] we approximate the above mentioned functionals by using more regular energies defined on Sobolev spaces. The approximation takes place in the Γ-convergence sense and it is carried out both via continuous and discrete models.

All the results contained in the papers quoted above are collected with simplified proofs and further corollaries in my PhD thesis [7].

References [9],[11],[14],[19]: Recently my attention has been drawn to the investigation of the asymptotic behaviour of obstacle problems for free-discontinuity functionals in perforated domains. These energies may be involved in the homogenization of brittle particle-reinforced composites and brittle porous bodies.

For bodies with a periodic distribution of rigid reinforcements, one may be interested in analyzing the behaviour of the (say) elastic energy as the diameter of the reinforcements vanishes. In the case of generalized anti-plane shear and by selecting the Mumford-Shah energy as a prototype, one analyzes the asymptotics as ε tends to 0 of

$$\mathcal{F}_\varepsilon(u) = \int_\Omega |\nabla u|^2\, dx + \mathcal{H}^{n-1}(S_u) \quad u \in SBV(\Omega),\, u = 0\, \mathcal{L}^n \text{ a.e. on } \mathbf{E}_\varepsilon$$

$+\infty$ otherwise in $SBV(\Omega)$, where $\mathbf{E}_\varepsilon = \Omega \cap \cup_{\underline{i} \in \mathbf{Z}^n}(\varepsilon \underline{i} + r_\varepsilon E)$, with E a given set in the reference cell $[-1/2, 1/2)^n$.

In [9] we identify the relevant scaling for r_ε as $\varepsilon^{\frac{n}{n-1}}$ and prove in such a regime the Γ-convergence of the family $(\mathcal{F}_\varepsilon)$ to a limit energy containing an extra term which is a finite penalization keeping track of the local capacity density of the homogenizing obstacles.

Moreover, we study an equivalent formulation of the obstacle problem for reference perforations with Lebesgue measure zero. Further results and generalizations are also presented.

In [11] we characterize the lower semicontinuous envelope of Mumford-Shah type energies with pure obstacles, and show that it can be written by means of an $(n-1)$-dimensional geometric variational measure introduced by De Giorgi in his theory of non parametric minimal surfaces with obstacles.

In [14] we provide a variational model for hyperelastic brittle porous media by studying the asymptotic behaviour in terms of Γ-convergence as $\varepsilon \to 0^+$ of

$$\mathcal{F}_\varepsilon(u) = \int_{\Omega_\varepsilon} |\nabla u|^2\, dx + \mathcal{H}^{n-1}(S_u), \qquad u \in L^1(\Omega) \cap SBV(\Omega_\varepsilon),$$

$+\infty$ otherwise in $L^1(\Omega)$. Here, Ω_ε represents the porous body and it is is given by $\Omega \setminus \cup_{\underline{i}\in\mathbf{Z}^n}\varepsilon(\underline{i}+E)$, with E a given compact set in the reference cell $[-1/2, 1/2)^n$.

As a first non-trivial step we show that the domain of any limit functional is $SBV(\Omega)$, despite the degeneracies introduced by the perforations. To this aim we provide an improved version of De Giorgi' SBV Poincaré-Wirtinger type inequality in two dimensions.

Then we show explicit formulas both for the bulk and surface energy densities of the Γ-limit, representing in our model the effective elastic and brittle properties of the porous medium, respectively. Further results and generalizations are also presented.

In [19] we develop an alternative method to deal with the homogenization of Neumann problems for free-discontinuity energies defined on perforated domains studied in [14]. The new ideas we introduce are such that we extend the results obtained in [14] for what the regularity and the location of the holes are concerned.

In addition, we improve previous contributions for the homogenization of degenerate energies in the Sobolev setting, too.

Reference [10]: We examine the asymptotic behaviour in terms of Γ-convergence of the following one dimensional energy

$$F_\varepsilon(u) = \mu_\varepsilon \int_I \int_I \frac{|u(x)-u(y)|^2}{|x-y|^2}\, dx\, dy + \eta_\varepsilon \int_I W\left(\frac{u(x)}{\varepsilon}\right) dx$$

where I is a given interval, W is a one-periodic potential that vanishes exactly on $\mathbf{Z}$. Different regimes for the asymptotic behaviour of the parameter μ_ε and η_ε are considered. In a very diluted regime we get a limit defined on $BV(I)$ and proportional to the total variation of u. In this particular case we also consider the limit of a suitable boundary value problem for which we characterize the second order Γ-limit.

The study under consideration is motivated by the analysis of a variational model for a very important class of defects in crystals, the dislocations, and the derivation of macroscopic models for plasticity.

References [12],[13]: A *variational integrator* is any recursive rule that allows one to calculate discrete trajectories from initial data and coincides with the discrete Euler-Lagrange equation of some discrete (or discretized) Lagrangian. It is a discrete scheme for evaluating (numerically) the dynamics described by some Hamilton or d'Alembert-Lagrange principle.

In particular, when the potential appearing in the Lagrangian functional can be additively subdivided in subsystems, e.g. distinct groups of particles or finite elements in space in case of continuous problems, it is possible to select asynchronous time discretizations. Asynchronous variational integrators (AVIs) then follow. They preserve the symplectic structure of the original continuum system. Thus, for non-constant time steps the energy evaluated numerically oscillates around its average value.

In [12] we prove that within the setting of linear elastodynamics of simple bodies the discrete Lagrangians obtained by following the scheme of asynchronous variational integrators converges in time to a partially discretized Lagrangian, continuos in time, the convergence of which in space is assured by standard arguments when the finite element mesh is progressively refined. In doing that we follow the Γ-convergence argument provided by Müller and Ortiz in the one-dimensional scalar case for the elementary oscillator.

In [13] we start the analysis of variational integrators (in the synchronous and asynchronous versions) for the linear dynamics of complex bodies, those bodies for which the material substructure prominently influence the gross behavior. We consider cases in which the material substructures display peculiar inertia (additional to the macroscopic one) and internal dissipation. The dynamics of such bodies is governed by a d'Alembert-Lagrange-type principle. We do no treat any specific model of complex materials. We develop our analyses in the general model-building framework of the mechanics of complex bodies.

The technique that we develop is new: it is based essentially on BV estimates on the time-rates of the fields involved. In turn these estimates let us gain strong compactness properties and permits us to pass to the limit directly into the discrete stationarity equations. This method is alternative to the ones already developed to study the convergence properties of variational integrators in literature. Furthermore, the BV estimates are instrumental to treat dissipative or forced systems.

References [15],[17],[20]: Γ-convergence methods are used to prove homogenization results for fractional obstacle problems in perforated domains.

In [15] we consider a framework introduced by Caffarelli and Mellet (see refs. [6],[7]) in which the obstacles have random sizes and shapes and their capacity scales according to a stationary and ergodic process. By means of a trace-like representation of fractional Sobolev norms in terms of weighted Sobolev energies established by Caffarelli and Silvestre (see ref. [8] in the bibliography), a weighted ergodic theorem and a joining lemma in varying domains reminescent of a method introduced by De Giorgi on the way to match boundary conditions by increasing the energy only up to a small error we determine explicitely the limit energy. Our proof is alternative to those proposed by Caffarelli and Mellet.

The previous result is further extended in [17] where we develop an intrinsic point avoiding the above mentioned extension result. Furthermore, our analysis is performed in a rather general aperiodic setting defined after Delone set of points. In addition, we consider two random settings: obstacles with random sizes and shapes located on points of a standard lattice, and the case of random homothetics obstacles centered on random Delone sets of points.

In [20] we consider the same problem in a vector-valued setting in view of applications to phase field models for ferroelectric solids and dislocations. In particular, we analyze in details the scaling-invariant case, since the subcritical one can be deduced by the analysis of the latter together with the results in [17].

Reference [16]: We study a 1-capacitary type problem in $\mathbf{R}^2$: given a set E, we minimize the perimeter (in the sense of De Giorgi) among all the sets containing E (modulo $\mathcal{H}^1$) and satisfying an indecomposability constraint (according to the definition by Ambrosio, Caselles, Masnou and Morel see ref. [1] in the bibliography). By choosing suitably the representant of the relevant set E, we show that a convexification process characterizes the minimizers.

As a consequence of our result we determine the 1-capacity of (a suitable representant of) sets with finite perimeter in the plane.

References [18],[21]: We consider a classical problem in Calculus of Variations: That of establishing and characterizing the weak lower semicontinuity of integral functionals via properties enjoyed by the energy densities. The functionals we take into account are defined on Sobolev maps taking values into the space of Almgren's Q-valued functions in [18], while in [21] into manifolds.

In both cases we are able to characterize lower semicontinuous functionals by means of a suitable notion of quasi-convexity, what is more relevant is that our arguments are intrinsic, contained within the theory of metric space valued Sobolev maps. We do not use embeddings in Euclidean spaces.

In addition, in [18] we recover earlier results by Mattila (see ref. [10] in the bibliography there), and we answer to a question raised by Mattila himself in the same paper.

Reference [22],[23]: In these two papers we study some regularity properties of local minimizers of the Mumford-Shah energy in dimension two.

More precisely, in [22] we are concerned with low regularity results establishing the equivalence of the strong and weak (more precisely SBV) formulations of the problem. By means of a direct variational argument we find an explicit energy-threshold criterion selecting regular points of local minimizers of the Mumford-Shah energy in dimension two. From this, we find an explicit, though not optimal, constant for the density lower bound estimate for points in the closure of the jump set in a way that we can infer the essential closure of the latter. The latter result were proved with different methods in the celebrated papers by De Giorgi, Carriero and Leaci, and Dal Maso, Morel and Solimini.

In [23] we establish the validity of a conjecture proposed by De Giorgi concerning the higher integrability of the approximate gradients for local minimizers of the Mumford-Shah energy in dimension two.

Following Ambrosio, Fusco and Hutchinson, such a property implies an estimate on the Hausdorff dimension of the singular set of the set of discontuities of any local minimizer. In addition, we provide an energetic condition equivalent to the validity of the Mumford-Shah conjecture.

Reference [24]: This paper is a survey paper on De Giorgi's Γ-convergence, a variational theory modelled upon the convergence of families of (perturbed) minimum problems and of the corresponding minimizers.

After reviewing briefly the basic theory in a friendly way and accounting for several recent new insights, we discuss three examples of static mechanical models which can be analyzed by means of Γ-convergence arguments.