Testi di matematica in italiano e in inglese:

esperienze a confronto

Integration 1

Differentiation Reversed

When x^2 is differentiated with respect to x the derivative is $2x$.
Conversely, if the derivative of an unknown function is $2x$ then it is clear that the unknown function could be x 2.
This process of finding a function from its derivative, which reverses the operation differentiating, is called *integration*.

The Constant of Integration

As seen above, $2x$ is the derivative at x^2, but *it* is also the derivative at $x^2 + 3$, and, in fact, the derivative of x^2 + any constant.
Therefore the result of integrating $2x$, which is called *the integral of 2x*, is *nat* a function but is of the form

$x^2 + K$ where K is any constant

K is called *the constant of integration*.

This is written J $2x$ dx = $x^2 + K$

where J ... dx means 'the integral of ... w.r.t. x'.

Integrating *any* function reverses the process of differentiating so, for any function ; we have

J !f(x) dx = f(x) + K

e.g. because differentiating x^3 w.r.t. x gives $3x^2$ we have J $3x^2\,dx = x^3 + K$ and it follows that J $x^2\,dx$ = tx3 + K

Note that it is not necessary to write tK in the second form, as K represents *any* constant in either expression.
In general, the derivative of *xn+l* is (n + *1)xn* so J x" *dx* = n!l *xn+l* +K

i.e. to integrate a power of x, increase that power by 1 and *divide* by new power.

This rule can be used to integrate any power of x *except* -1, which is considered later ..

We know that $\frac{d}{dx} e^x = e^x$,

$$\int e^x \, dx = e^x + K$$

Integrating a Sum or Difference of Functions

We saw in Chapter 13 that a function can be differentiated term by term. Therefore, as integration reverses differentiation, integration also can be done term by term. To integrate products or quotients of functions, first express them as sums or differences of functions

e.g.

Find the integral of

$$\int \left(1 + x^7 + \frac{1}{x^2} - \sqrt{x}\right) dx = \int (1 + x^7 + x^{-2} - x^{1/2})\, dx$$

$$= \int 1\, dx + \int x^7\, dx + \int x^{-2}\, dx - \int x^{1/2}\, dx$$

$$= x + \frac{1}{8}x^8 + \frac{1}{-1}x^{-1} - \frac{1}{\frac{3}{2}}x^{3/2} + K$$