A modular formalization of bicategories in type theory *

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Abstract

We report on our ongoing effort to implement a library of bicategories in type theory, specifically in the UniMath library of univalent mathematics. It is developed in the context of a larger project aimed at defining signatures for dependent type theories and their models.

1 Introduction

Our goal is to study the categorical semantics of dependent type systems, in univalent type theory. A crucial step in this project is to set up a formal implementation of the fundamental category-theoretic definitions. Due to the complexity of the objects under consideration, it is of fundamental importance that constructions and reasoning can be carried out in a highly modular way. In this work, we present our attempt at developing such a modular formalization.

2 Background

The UniMath library of univalent mathematics [5] currently contains quite a few results on 1-category theory, but the theory of higher categories, in particular, of bicategories, is not well-developed yet.

Specifically, one development of bicategories in UniMath has been contributed by Mitchell Riley[1]. Riley gives the definition of "univalent bicategories", and shows that an equivalence of univalent categories induces an identity between them—a first step towards showing that the bicategory of univalent categories is univalent. However, his definition of bicategories—in the style of enrichment in 1-categories—seems not amenable to modular reasoning on these complex gadgets. Indeed, defining the hom-objects to be categories mixes data and properties in a way that makes it difficult to use in a proof-relevant setting.

In this work, we present an alternative formalization of bicategories that avoids this problem. Additionally, it allows for modular construction of bicategories of complex objects using a bicategorical version of displayed categories by Ahrens and Lumsdaine [1].

3 Bicategories

The prevalent definition of bicategories in the literature, in the style of weak enrichment over 1-categories, is very concise and easy to check for correctness.

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[1] https://github.com/UniMath/UniMath/pull/409

Here, we follow an alternative presentation [4] where the structure of 0-cells (objects), 1-cells (morphisms), and 2-cells is made explict. This approach, in contrast to the former, adheres intrinsically to a general established design principle in intensional mathematics of strictly separating data and properties.

4 Displayed bicategories and modularity

Ahrens and Lumsdaine's displayed categories [1] allow for a modular construction of complex 1-categories from simpler ones, in "layers". A prototypical example comes from the stratification of structures in algebra. For instance, the construction of the category of groups from the category of sets plus some extra structure can easily be factorized and expressed via the "displayed group structure" over the displayed monoid structure on the category of sets.

Following this pattern, we give the bicategorical variant of displayed categories, and arrange the displayed 1-categories into a displayed bicategory over the bicategory of categories. We then systematically use displayed bicategories to implement several bicategorical constructions. Some currently implemented examples include direct product, sigma structures, and functor and cofunctor categories.

These constructions were enough to build in UniMath, in a modular way, the bicategory CwF of categories with families [2], in the reformulation by Fiore [3, Appendix].

5 Conclusions

Our code is available from the UniMath repository[2]. It consists of about 4,000 lines of code.

We are planning to extend our "zoo" of constructions of displayed bicategories. In particular, we are going to study univalent bicategories and modular ways of showing that a given bicategory, constructed from a displayed one, is univalent. Our project on signatures for dependent type theories will provide a challenging test-bed for the usability of our library.

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[2] `https://github.com/UniMath/UniMath/pull/925`