Generalized mean value property for caloric functions

by Henrik Shahgholian
(The Royal Institute of Technology, Stockholm, Sweden)

The mean-value property for harmonic functions

$$\int_{B_1(0)} h(x)\ dx = C\ h(0),$$

is a well-known fact. A similar phenomenon for caloric functions

$$\iint_{\Omega} h(x,t)\ dxdt = C\ h(0,0),$$

is as well as unknown, i.e., it is unkwon whether one can find $\Omega \subset \mathbb{R}^{n+1}$ such that the above integral identity holds for all functions $h \in L^1(\Omega)$ such that $\Delta h - D_t h = 0$ in Ω.

Although, with some efforts, using variational inequalities, one can prove the existence of a bounded domain $\Omega \subset \mathbb{R}^{n+1}$ with the above mean-value property for caloric functions, it seems that this type of question is untouched in the literatures.

In this talk we will discuss different aspects of a generalized form of this problem. Namely the problem of finding a bounded $\Omega \subset \mathbb{R}^{n+1}$ such that

$$\iint_{\Omega} h(x,t)\ dxdt = \iint h(x,t) g(x,t)\ dxdt,$$

where g is a given function, and Ω is such that the support of g is in Ω.

The particular case of g being the Dirac measure at the origin gives the mean-value property for caloric functions.